

Muon decay in orbit spectra for $\mu - e$ conversion experiments

Andrzej Czarnecki · Xavier Garcia i Tormo ·
William Joseph Marciano

Published online: 3 January 2012
© Springer Science+Business Media B.V. 2011

Abstract We have determined in detail the electron spectrum in the decay of bound muons. These results are especially relevant for the upcoming $\mu - e$ conversion experiments.

Keywords Muon decay · Muonic atoms · Electron spectrum · Muon-electron conversion

1 Introduction

From the observation of neutrino oscillations, we now know that lepton flavors are not conserved. However, the mixing and small neutrino mass differences seen in oscillations have a negligible effect on charged-lepton flavor violating (CLFV) reactions. Thus, the CLFV reactions provide a discovery window for interactions beyond Standard Model expectations [1, 2].

Muons play a central role in searches for CLFV [1, 2], because they can be produced in large numbers and live relatively long. One reaction that can be probed with particularly high sensitivity is coherent muon-electron conversion in a muonic atom,

$$\mu^- + (A, Z) \rightarrow e^- + (A, Z), \quad (1)$$

where (A, Z) represents a nucleus of atomic number Z and mass number A . It has the advantage of producing just a single particle, a mono-energetic electron. It does

A. Czarnecki (✉) · X. Garcia i Tormo
Department of Physics, University of Alberta, Edmonton, AB, Canada T6G 2E1
e-mail: andrzejc@ualberta.ca

X. Garcia i Tormo
Institut für Theoretische Physik, Universität Bern, Sidlerstrasse 5, 3012 Bern, Switzerland

W. J. Marciano
Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, USA

not have the problem of accidental background that plagues searches for the decay $\mu^+ \rightarrow e^+ \gamma$, which can be mimicked by a positron from a normal muon decay and a photon coming from the radiative decay of a different muon, bremsstrahlung, or positron annihilation-in-flight. Various experiments have been performed over the years to search for the conversion [3]. The most stringent results come from the SINDRUM II Collaboration [4], which reports an upper limit of 7×10^{-13} for the branching ratio of the conversion process relative to muon capture in gold. Several new efforts are being planned. In the nearest future, the DeeMe Collaboration [5] has proposed to reach 10^{-14} sensitivity. Larger scale searches, Mu2e at Fermilab [6] and COMET at J-PARC [7], aim for sensitivities below 10^{-16} . In the long run, intensity upgrades at Fermilab and the proposal PRISM/PRIME at J-PARC may allow them to reach 10^{-18} sensitivity. A quite remarkable improvement of about four orders of magnitude, with respect to the current limit, is therefore envisaged.

The success of the conversion searches depends critically on control of the background events. The signal for the $\mu - e$ conversion process in (1) is a monoenergetic electron with energy $E_{\mu e}$, given by

$$E_{\mu e} = m_\mu - E_b - E_{\text{rec}}, \quad (2)$$

where m_μ is the muon mass, $E_b \simeq Z^2 \alpha^2 m_\mu / 2$ is the binding energy of the muonic atom, and $E_{\text{rec}} \simeq m_\mu^2 / (2m_N)$ is the nuclear-recoil energy, with α the fine-structure constant and m_N the nucleus mass. The main physics background for this signal comes from the so-called muon decay in orbit (DIO), a process in which the muon decays in the normal way, $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$, while in the orbit of the atom. Whereas in a free muon decay, in order to conserve energy and three-momentum, the maximum electron energy is $m_\mu / 2$, for DIO, the nucleus recoil can balance the electron's three-momentum taking basically no energy. This allows for the maximum electron energy to be $E_{\mu e}$, close to the full muon mass m_μ . Therefore, the high-energy electrons from the muon decay in orbit constitute a background for conversion searches.

2 Muon decay in orbit

Several theoretical studies of the muon decay in orbit have been published. Expressions describing the electron spectrum including relativistic effects in the muon wavefunction, the Coulomb interaction between the electron and the nucleus and a finite nuclear size have been available for some time [8–10]. However, the high-energy endpoint of the spectrum, which is the most important region for conversion-search experiments, was not studied in detail. Shanker [11, 12] did study the high-energy end of the electron spectrum, and presented approximate results which allow for a quick rough estimate of the muon decay in orbit contribution to the background in conversion experiments. We have performed a new evaluation of the DIO spectrum, considering in detail all the effects needed in the high-energy region [13]. Our results describe the background contribution for $\mu - e$ conversion searches, as well as a check on previous low- and high-energy partial calculations [10, 11] and an interpolation between them. It is worth emphasizing that not only the high-energy region is relevant for conversion experiments, but the full spectrum is necessary in order to study reconstruction errors in the detector.

Table 1 Values for muon energies E_μ , nuclear masses m_N , and endpoint energies $E_{\mu e}$

Nucleus	Z	E_μ (MeV)	m_N (MeV)	$E_{\mu e}$ (MeV)
C	6	105.557	11,188	105.06
Al	13	105.194	25,133	104.973
Si	14	105.121	26,162	104.91
Ti	22	104.394	44,588	104.272

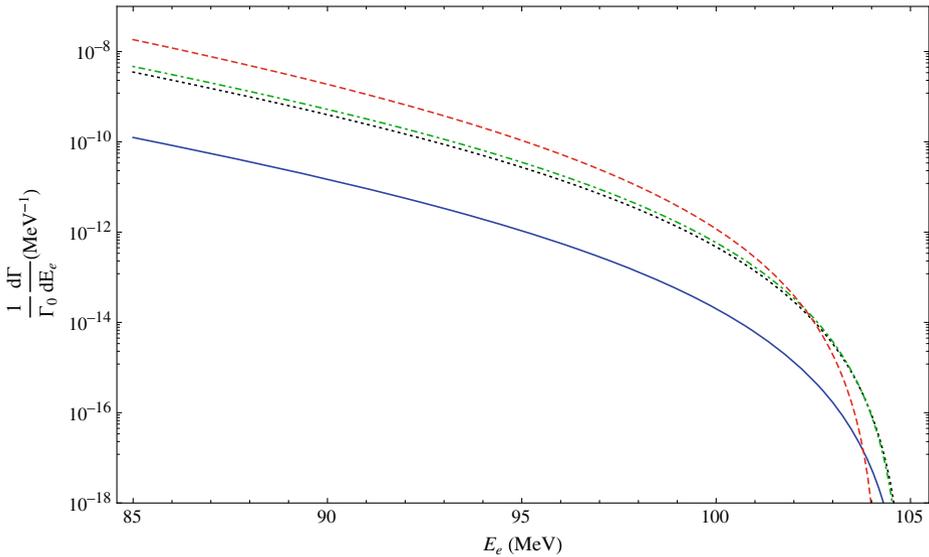


Fig. 1 Electron spectrum, normalized to the free-muon decay rate Γ_0 . The *solid blue line* is for carbon, the *black dotted line* for aluminum, the *green dot-dashed line* for silicon and the *red dashed line* for titanium

To obtain the correct result for the high-energy tail of the spectrum it is crucial to include nuclear-recoil effects, since they modify the endpoint energy (see (2)). Also, to produce an on-shell electron with energy around m_μ , either the muon must be at the tail of the bound-state wavefunction or the produced electron must interact with the nucleus. This tells us that the full Dirac equation for the muon as well as the interaction of the outgoing electron with the field of the nucleus must be taken into account. Also, finite-nuclear-size effects will be most important in this region. Order α radiative corrections are not expected to significantly modify the results at the endpoint, and are not included in our results. Uncertainties in the modelling of finite nuclear-size effects induce errors in the spectrum that increase as we approach the endpoint, but those errors are never larger than a few percent.

3 Results and discussion

Here we present our results for the elements that are relevant for the upcoming conversion experiments. The DeeMe Collaboration plans to use a silicon-carbide target, whereas the Mu2e and COMET Collaborations are considering aluminum

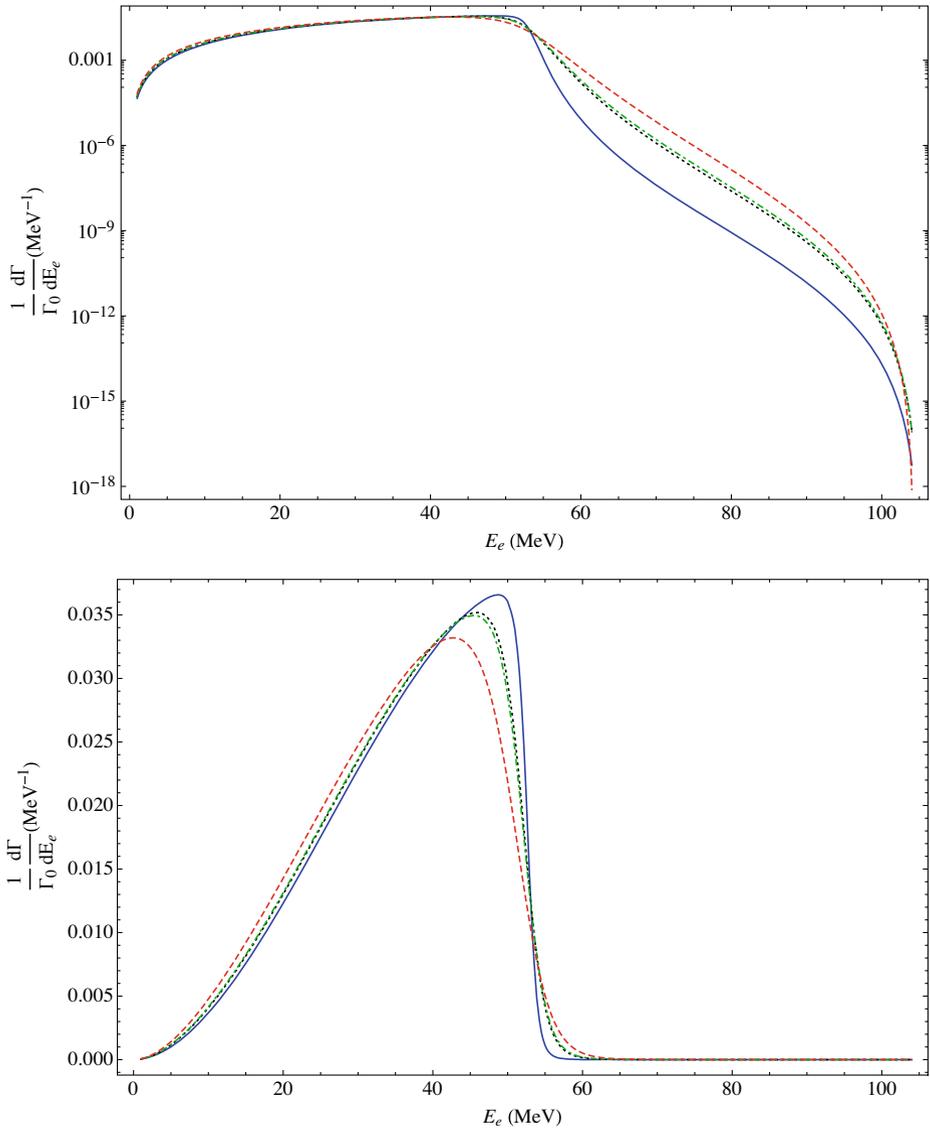


Fig. 2 Electron spectrum for the full range of E_e ; see caption of Fig. 1

and titanium as targets. In Table 1 we give the values of the bound muon energy $E_\mu = m_\mu - E_b$ and the electron endpoint energy $E_{\mu e}$, for carbon, aluminum, silicon and titanium.

We present the results of the numerical evaluation of the spectra for those elements in Figs. 1 and 2. Regarding the nuclear distributions, a two-parameter Fermi distribution has been used for aluminum and titanium, a three-parameter Fermi distribution for silicon and a Fourier–Bessel expansion for carbon [13–15]. Those results are useful for assessing the DIO background events from carbon for

the DeeMe experiment which will search primarily for conversion in silicon. They also illustrate the electron resolution requirements, as a function of stopping target, needed to reach future high sensitivity goals.

The high sensitivity that the upcoming conversion experiments will reach may also allow them to improve the present bounds on some exotic muon decays, like the decay of the muon into an electron and a majoron (a Goldstone boson that appears in models where lepton number is a spontaneously broken global symmetry) [16].

Acknowledgements This research was supported by Science and Engineering Research Canada (NSERC) and by the United States Department of Energy under Grant Contract DE-AC02-98CH10886.

References

1. Kuno, Y., Okada, Y.: Rev. Mod. Phys. **73**, 151 (2001). [arXiv:hep-ph/9909265](#)
2. Czarnecki, A., Marciano, W.J.: In: Roberts, B.L., Marciano, W.J. (eds.) Lepton Dipole Moments, p. 11. World Scientific, Singapore (2009); Adv. Ser. Dir. HEP, vol. 20; Kuno, Y.: *ibid.*, p. 701; Okada, Y.: *ibid.*, p. 683
3. Marciano, W.J., Mori, T., Roney, J.M.: Annu. Rev. Nucl. Part. Sci. **58**, 315 (2008)
4. Bertl, W.H., et al., SINDRUM II Collaboration: Eur. Phys. J. C **47**, 337 (2006)
5. Aoki, M.: PoS **ICHEP 2010**, 279 (2010)
6. Carey, R.M., et al., Mu2e Collaboration: Proposal to search for $\mu^- N \rightarrow e^- N$ with a single event sensitivity below 10^{-16} . Fermilab Proposal 0973 (2008)
7. Cui, Y.G., et al., COMET Collaboration: Conceptual design report for experimental search for lepton flavor violating $\mu^- - e^-$ conversion at sensitivity of 10^{-16} with a slow-extracted bunched proton beam (COMET). KEK Report 2009–2010 (2009)
8. Haenggi, P., Viollier, R.D., Raff, U., Alder, K.: Phys. Lett. **B51**, 119 (1974)
9. Watanabe, R., Fukui, M., Ohtsubo, H., et al.: Prog. Theor. Phys. **78**, 114 (1987)
10. Watanabe, R. et al.: At. Data Nucl. Data Tables **54**, 165 (1993)
11. Shanker, O.U.: Phys. Rev. **D25**, 1847 (1982)
12. Shanker, O.U., Roy, R.: Phys. Rev. **D55**, 7307 (1997)
13. Czarnecki, A., Garcia i Tormo, X., Marciano, W.J.: Phys. Rev. **D84**, 013006 (2011). [arXiv:1106.4756](#) (hep-ph)
14. De Vries, H., De Jager, C.W., De Vries, C.: At. Data Nucl. Data Tables **36**, 495–536 (1987)
15. Fricke, G., Bernhardt, C., Heilig, K., Schaller, L.A., Schellenberg, L., Shera, E.B., De Jager, C.W.: At. Data Nucl. Data Tables **60**, 177 (1995)
16. Garcia i Tormo, X., Bryman, D., Czarnecki, A., Dowling, M.: Phys. Rev. **D84**, 113010 (2011). [arXiv:1110.2874](#) (hep-ph)