Noise analysis in MOSFET Preamplifiers

(Revised on November 8, 2006 by E. Won)

I. ANALYSIS BASED ON NIM A288 (1990) 157

We discuss the noise from the MOSFET based preamplifier-shapers in this note. In the field of particle physics experiment, it is common that the noise of a charge amplification electronics is expressed in equivalent noise charge (ENC) which is rms of the pedestal in the number of electrons. According to reference [1], the noise may be expressed as

$$\mathrm{ENC}/C_{\mathrm{det}} = \left[(e^4/64)4kT(2/3)\Gamma/g_m\tau\right]^{\frac{1}{2}} \tag{1}$$

where

$$\begin{cases} e & = 2.718 \\ k & = 1.38 \times 10^{-23}\ \mathrm{J/K} \quad \text{(Boltmann constant)} \\ T & = \text{temperature} \\ \Gamma & = \text{excess noise factor depending on channel length and source-drain voltage} \\ g_m & = \text{transconductance of the low-noise p-channel device} \\ \tau & = \text{filter time constant of the } CR - RC \text{ filter.} \end{cases}$$

This expression appears to be the channel noise term in reference [2], as it includes the transconductance g_m in the denominator. If you assume $g_m = 1.4$ mA/V ($I_{\mathrm{bias}} = 50\ \mu$A) and $\Gamma = 1.4$ at the room temperature $T = 300$ K, the numerical value becomes

$$\begin{aligned} \mathrm{ENC}/C_{\mathrm{det}} &= \Big[2.718^4/64 \cdot 1.38 \times 10^{-23}\ \mathrm{J/K} \cdot 300\ \mathrm{K} \\ &\times\ (2/3) \cdot 1.4/(1.4\ \mathrm{mA/V} \cdot 320\ \mathrm{ns})\Big]^{\frac{1}{2}} \\ &= 0.54 \times 10^{-5}\ \mathrm{C/F} \\ &= 0.0054\ \mathrm{fC/pF} \end{aligned} \tag{2}$$

where the experiment shows that the $\mathrm{ENC}/C_{\mathrm{det}} = 33$ electrons or 0.005 fC per pF, and agrees fairly well with the numerical value from the expression in Eq. (1).

II. ANALYSIS BASED ON NIM A301 (1991) 506

In this paper, authors argued that the total noise of the system without considering the associated detector bias resistor and leakage current is dominated only by the noise of the input transistor. The noise is then caused by three different independent mechanisms:

Flicker noise ($S_{1/f}$): $1/f$ noise behavior.

Channel noise (S_{ct}): white noise behavior.

Bulk-resistance noise (S_{br}): white noise behavior.

When referred to the gate, their respective noise voltage spectral density can be expressed as:

$$\begin{aligned} S_{1/f}(\omega) &= \frac{2\pi F_k}{WL_{\text{eff}}\,\omega}\ [V^2/(rad/s)] \\ S_{ct}(\omega) &= \frac{4\Gamma(\eta+1)kT}{3g_m\pi}\ [V^2/(rad/s)] \\ S_{br}(\omega) &= \frac{2R_{\text{bulk}}\,\eta^2 kT}{\pi}\ [V^2/(rad/s)] \end{aligned} \tag{3}$$

where:

W	is the transistor width,
L_{eff}	is the effective transistor length
F_k	is a process-dependent constant
η	is the ratio between the bulk-to-channel and gate-to-channel transconductances
R_{bulk}	is the bulk resistance.

If we combine all terms to get ENC, we get

$$\frac{\text{ENC}}{C_t} = \frac{e}{q}\left[\frac{F_k}{2WL_{\text{eff}}} + \frac{\Gamma(\eta+1)kT}{3g_m T_p} + \frac{R_{\text{bulk}}\,\eta^2 kT}{2T_p}\right]^{1/2}, \tag{4}$$

where q is the electon charge. In the paper, $WL_{\text{eff}} = 4300\ \mu\text{m} \times 0.8\ \mu\text{m}$, $g_m = 3$ mA/V, $F_k = 7.6 \times 10^{-22}\ \text{m}^2\text{V}^2$, $\Gamma = 2$, $\eta = 0.15$, $R_{\text{bulk}} = 2\ \text{k}\Omega$, and $T_p = 1500$ ns are used. There is one issue to be addressed. In reference [2], it appears the that the Flicker noise constant F_k is a dimensionless constant. If that is used, then the first term in Eq. (4) would have incorrect physical dimension. That could be checked with the reference [3], page 86. One can explicitly show that the Flicker noise term should have the physical dimension of m^2V^2 using the Eq. (3.2-13) in the reference [3]. If we have all the numerical values in Eq. (4), we get

$$\begin{aligned} \frac{\text{ENC}}{C_t} &= \frac{e}{q}\left[\frac{7.6\times 10^{-22}}{2\cdot 4300\cdot 0.8\times 10^{-12}}\right. \\ &+ \frac{2\cdot(1.15)\cdot 1.38\cdot 300\times 10^{-23+3+9}}{3\cdot 3\cdot 1500} \\ &+ \left.\frac{2\cdot 0.15^2\cdot 1.38\cdot 300\times 10^{-23+3+9}}{3\cdot 3\cdot 1500}\right]^{1/2}\cdot\frac{\text{C}}{\text{F}} \end{aligned} \tag{5}$$

Now, from these numerical values, we immediately notice that the dominant source of the noise is from the channel noise. The Flicker noise term is suppressed due to Flicker noise constant with a large value of WL_{eff}. Bulk-resistance noise term is also suppressed (but less) due to the term η^2. If we obtain the numerical value of the noise,

$$
\begin{aligned}
\frac{\mathrm{ENC}}{C_t} &= \frac{2.718}{1.6 \times 10^{-}19\ \mathrm{C}} \left[\frac{7.6}{2 \cdot 4300 \cdot 0.8} \times 100 \right. \\
&+ \frac{2 \cdot (1.15) \cdot 1.38 \cdot 300}{3 \cdot 3 \cdot 1500} \times 10 \\
&+ \left. \frac{2 \cdot 0.15^2 \cdot 1.38 \cdot 300}{3 \cdot 3 \cdot 1500} \times 10 \right]^{1/2} \times 10^{-6} \cdot \frac{\mathrm{C}}{10^{12}\ \mathrm{pF}} \\
&\sim 15\ \frac{\text{electrons}}{\mathrm{pF}}.
\end{aligned} \tag{6}
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The paper quotes 16 /pF so the numerical calculation in this note is not too far off from the quoted value in reference [2] (Why is different anyway?). The KUPID design should be optimized for the noise.

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[1] E. Beuville, K. Border, E. Chesi, E. M.M. Heijne, P. Jarron, B. Lisowski, and S. Singh, Nucl. Instr. Meth. A **288**, 157 (1990).

[2] E. Nygard, P. Aspell, P. Jarron, P. Weihammer, and K. Yoshioka, Nucl. Instr. Meth. A **301**, 506 (1990).

[3] P. E. Allen and D. R. Holberg, CMOS Analog Circuit Design 2nd Edition (Holt, Rinehart and Winston, New York, 1987).