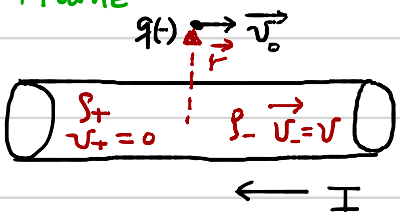
Special theory of relativity gives us length contraction and time dilation effect.

$$L = L_0\sqrt{1 - v^2/c^2}\ , \quad \Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2}$$

L_0: length in rest frame

Δt_0: time difference in rest frame



S frame

ρ_- : charge density of electrons in wire

v_- : Speed of electrons when current flows

A negatively charged particle is moving to the right with $\vec{v}_0$

→ So the moving particle will experience Lorentz force

$$\vec{F} = q\vec{v}_0 \times \vec{B}$$

For given I, magnetic field at distance r is

$$B \cdot 2\pi r = \mu_0 I\ , \quad B = \frac{\mu_0 I}{2\pi r} \quad \text{or} \quad \frac{1}{\epsilon_0 \mu_0} = c^2, \quad \mu_0 = \frac{1}{\epsilon_0 c^2}$$

$$\therefore B = \frac{1}{2\pi\epsilon_0 c^2}\frac{I}{r}$$

So the force on the particle becomes

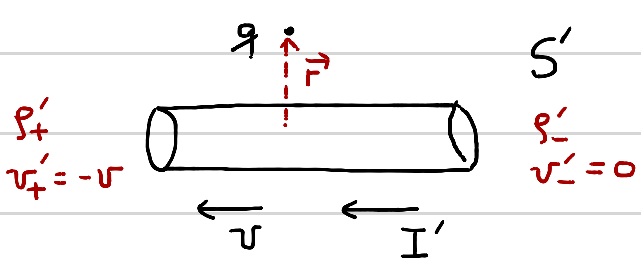
$$F = \frac{q v_0}{2\pi\epsilon_0} \cdot \frac{I}{c^2 r} \qquad I = \rho_- v A \quad (A : \text{area of wire})$$

$$= \frac{1}{2\pi\epsilon_0 c^2}\frac{q\rho_- A v v_0}{r}$$

Now let's assume $v = v_0$ for simplicity, so

$$F = \frac{q}{2\pi\epsilon_0}\frac{\rho_- A}{r}\frac{v^2}{c^2}$$ is obtained.

Now, we think of another reference frame S′ where the particle is at rest :



The particle is at rest. → No magnetic force!

Q: In S, force is magnetic, then what form of the force in S′?

L_0 : length of wire in the rest frame $\quad$ ρ_0 : charge density in the rest frame

The total charge Q is $Q = \rho_0 L_0 A_0$ $\quad$ A_0 : area of wire

· Q should be same for any frame

· The length of wire is shorter in moving frame : $L = L_0\sqrt{1-v^2/c^2}$

$$\rho_0 L_0 A_0 = \rho L_0\sqrt{1-v^2/c^2}\, A_0$$

$$\rho = \frac{\rho_0}{\sqrt{1-v^2/c^2}}$$ So the charge density of moving object is increased.

Let's apply it to our case. For (+) charge density,

$$\rho_+' = \frac{\rho_+}{\sqrt{1-v^2/c^2}}$$

And for (−), it is S' that has stationary charge density. So

$$\rho_- = \frac{\rho_-'}{\sqrt{1-v^2/c^2}} \quad \text{or} \quad \rho_-' = \rho_-\sqrt{1-v^2/c^2}$$

The net charge density in S' is

$$\rho' = \rho_+' + \rho_-' = \frac{\rho_+}{\sqrt{1-v^2/c^2}} + \rho_-\sqrt{1-v^2/c^2}$$

Since stationary wire is electrically neutral, $\rho_- = -\rho_+$

$$\rho' = \rho_+\left(\frac{1}{\sqrt{1-v^2/c^2}} - \sqrt{1-v^2/c^2}\right) = \rho_+\frac{v^2/c^2}{\sqrt{1-v^2/c^2}}$$

$\rho' \neq 0$ (while $\rho = \rho_+ + \rho_- = 0$)

Electric field at the distance r from axis of cylinder with density ρ':

$$E' \cdot 2\pi r h = \frac{1}{\epsilon_0} A h \rho'$$

$$E' = \frac{\rho' A}{2\pi\epsilon_0 r} = \frac{1}{2\pi\epsilon_0 r}\cdot\frac{\rho_+ A v^2/c^2}{\sqrt{1-v^2/c^2}}$$

So now we have electric force in S' :

$$F' = \frac{q}{2\pi\epsilon_0}\frac{\rho_+ A}{r}\frac{v^2/c^2}{\sqrt{1-v^2/c^2}}$$

(magnetic force was: $F = \frac{q}{2\pi\epsilon_0}\frac{\rho_- A}{r}\frac{v^2}{c^2}$)

$$\rightarrow F' = \frac{F}{\sqrt{1-v^2/c^2}}$$

So for the small velocities we have been considering, $F' \simeq F$.

→ For low velocities, we understand that magnetism and electricity are just two ways of looking at the same thing.

Q: What transverse momentum will the particle have after the force has acted for a little while? Same in S and in S'?

From $\vec{F} = \frac{d\vec{p}}{dt}$, $\Delta p_y = F\Delta t$ in S.

$\Delta p_y' = F'\Delta t'$ in S'

But in S', particle is at rest. So $\Delta t = \frac{\Delta t'}{\sqrt{1-v^2/c^2}}$

$$\therefore \quad \frac{\Delta p_y'}{\Delta p_y} = \frac{F'\Delta t'}{F\Delta t} = \frac{F/\sqrt{1-v^2/c^2} \cdot \Delta t\sqrt{1-v^2/c^2}}{F\Delta t} = 1$$

producing same physical result.