EM computation

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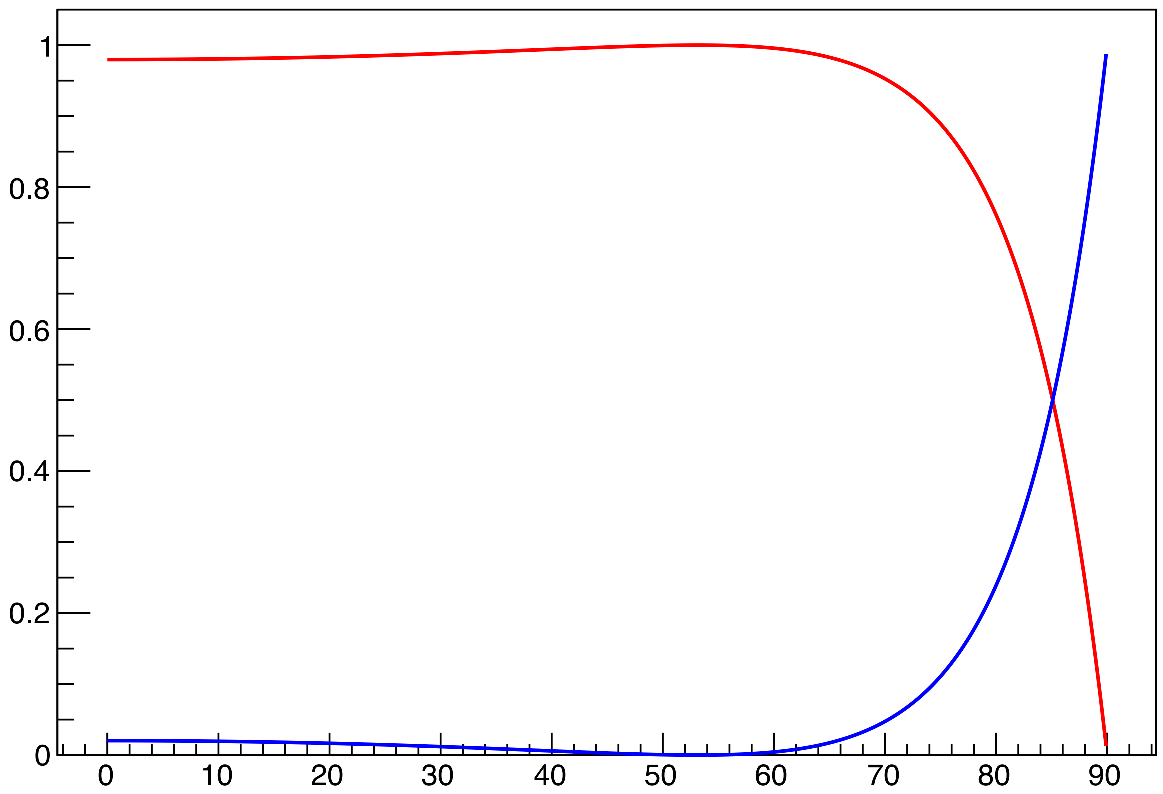
Transmission & Reflection of EM Wave

```
//
// Transmission & Reflection of EM wave (eunil@hep.korea.ac.kr)
//
// runs with root (http://root.cern.ch)
//
const Double_t n1 = 1.000277;   // index of refraction : air
const Double_t n2 = 1.3330;     // index of refraction : water
const Int_t nloop = 1000;

void trans()
{
  Double_t beta = n2/n1;
  Double_t x[nloop], T[nloop], R[nloop];

  for(Int_t i=0; i<nloop; i++)
  {
    Double_t rad = (1.0*i)/nloop*TMath::Pi()/2.0;
    Double_t deg = (1.0*i)/nloop*90.0;
    Double_t alpha = TMath::Sqrt(1.0-TMath::Power(TMath::Sin(rad)/n2,2))
                   / TMath::Cos(rad);

    x[i] = deg;
    T[i] = alpha*beta*TMath::Power(2.0/(alpha+beta),2);
    R[i] = TMath::Power((alpha-beta)/(alpha+beta),2);

  }

  TCanvas *cn = new TCanvas("cn","",200,10,700,500);

  gt = new TGraph(nloop,x,T); gt->SetLineColor(2); gt->SetLineWidth(2);
  gr = new TGraph(nloop,x,R); gr->SetLineColor(4); gr->SetLineWidth(2);
  TMultiGraph *mg = new TMultiGraph();
  mg->Add(gt);
  mg->Add(gr);
  mg->Draw("AC");
}
```

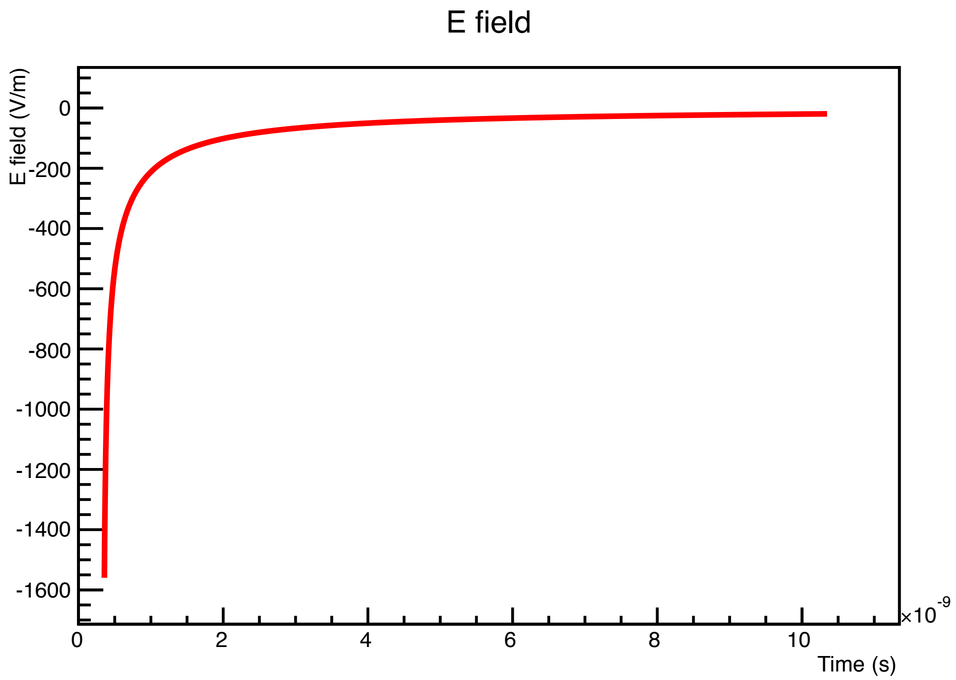
Fig. 9.17 of Griffiths

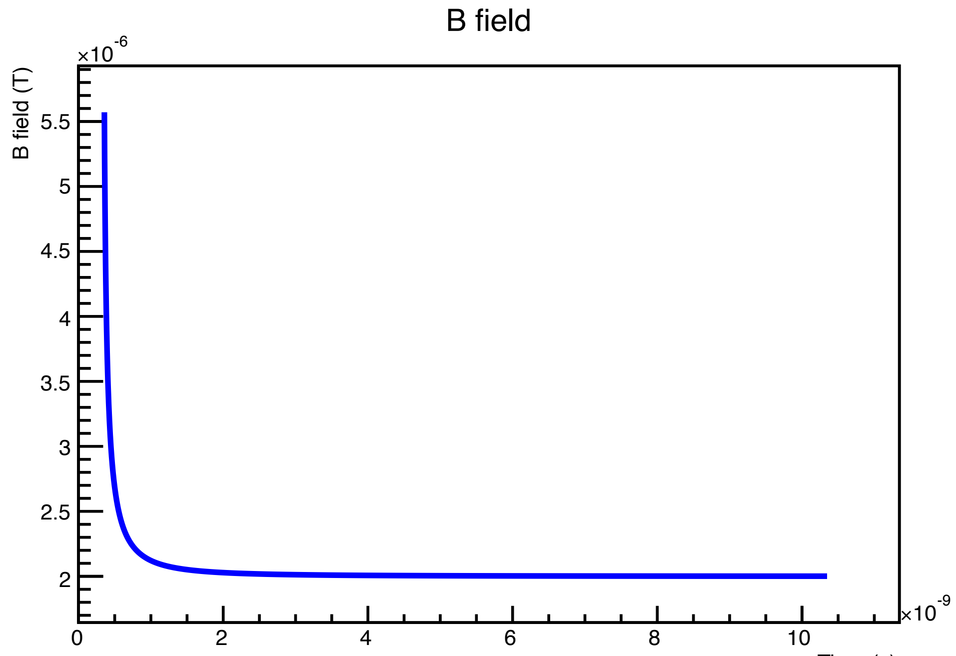


Retarded E and B fields-1

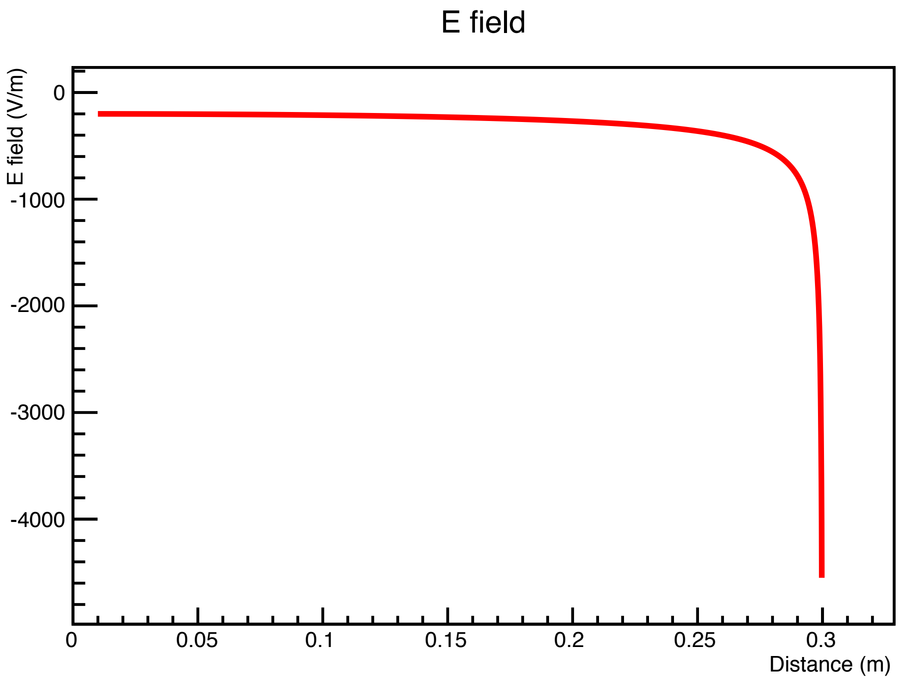
Example 10.2 gives $$\mathbf{E}(s,t) = -\frac{\mu_0 I_0 c}{2\pi\sqrt{(ct)^2 - s^2}}\hat{\mathbf{z}}$$ $$\mathbf{B}(s,t) = \frac{\mu_0 I_0}{2\pi s}\frac{ct}{\sqrt{(ct)^2 - s^2}}\hat{\phi}$$

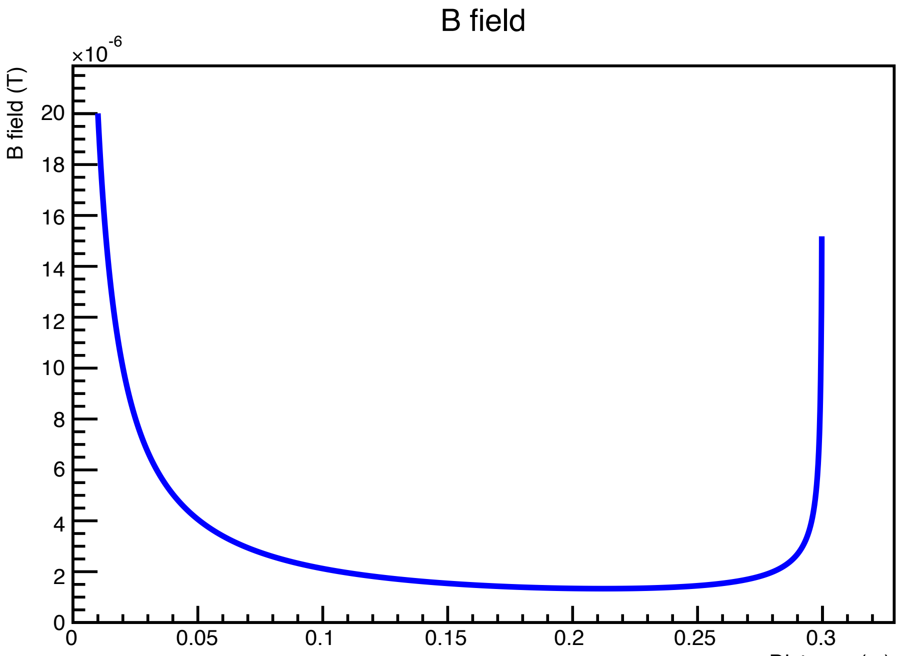
$I_0 = 1\ \mathrm{A}$ and at s = 0.1 m we get





and below are after 1 ns





Retarded E and B fields-2

With the code:

```
//
// E & B fields of a current carrying wire turned on at t=0
//
// runs with root (http://root.cern.ch)
//
const Double_t mu_0 = TMath::Pi()*4.0e-7;     // permeability in vacuum
const Double_t I_0  = 1.0;                    // current on wire (A)
const Double_t c    = 3.0e+8;                 // speed of light (m/s)
const Double_t t0   = 1.0e-9;                 // time from turn-on (s)
const Double_t ts   = 1.0e+1;                 // time span scale factor
const Double_t s0   = 1.0e-1;                 // location from wire (m)
const Double_t ss   = 2.9e+0;                 // location span scale factor

const Int_t nloop = 1000;

void curr()
{
  Double_t E_t[nloop], E_s[nloop], x[nloop], t[nloop];
  Double_t B_t[nloop], B_s[nloop];

  for(Int_t i=0; i<nloop; i++)
  {
    Double_t time = (1.0*i)/nloop*t0*ts + t0/2.8;
    Double_t xloc = (1.0*i)/nloop*s0*ss + 1.0e-2;

    t[i] = time;
    E_t[i] = -1.0*mu_0*I_0*c/(2.0*TMath::Pi()*TMath::Sqrt((c*time)*(c*time)-s0*s0));
    B_t[i] = +1.0*mu_0*I_0*c*time/(2.0*TMath::Pi()*s0*TMath::Sqrt((c*time)*(c*time)-s0*s0));

    x[i] = xloc;
    E_s[i] = -1.0*mu_0*I_0*c/(2.0*TMath::Pi()*TMath::Sqrt((c*t0)*(c*t0)-xloc*xloc));
    B_s[i] = +1.0*mu_0*I_0*c*t0/(2.0*TMath::Pi()*xloc*TMath::Sqrt((c*t0)*(c*t0)-xloc*xloc));
```

Retarded E and B fields-3

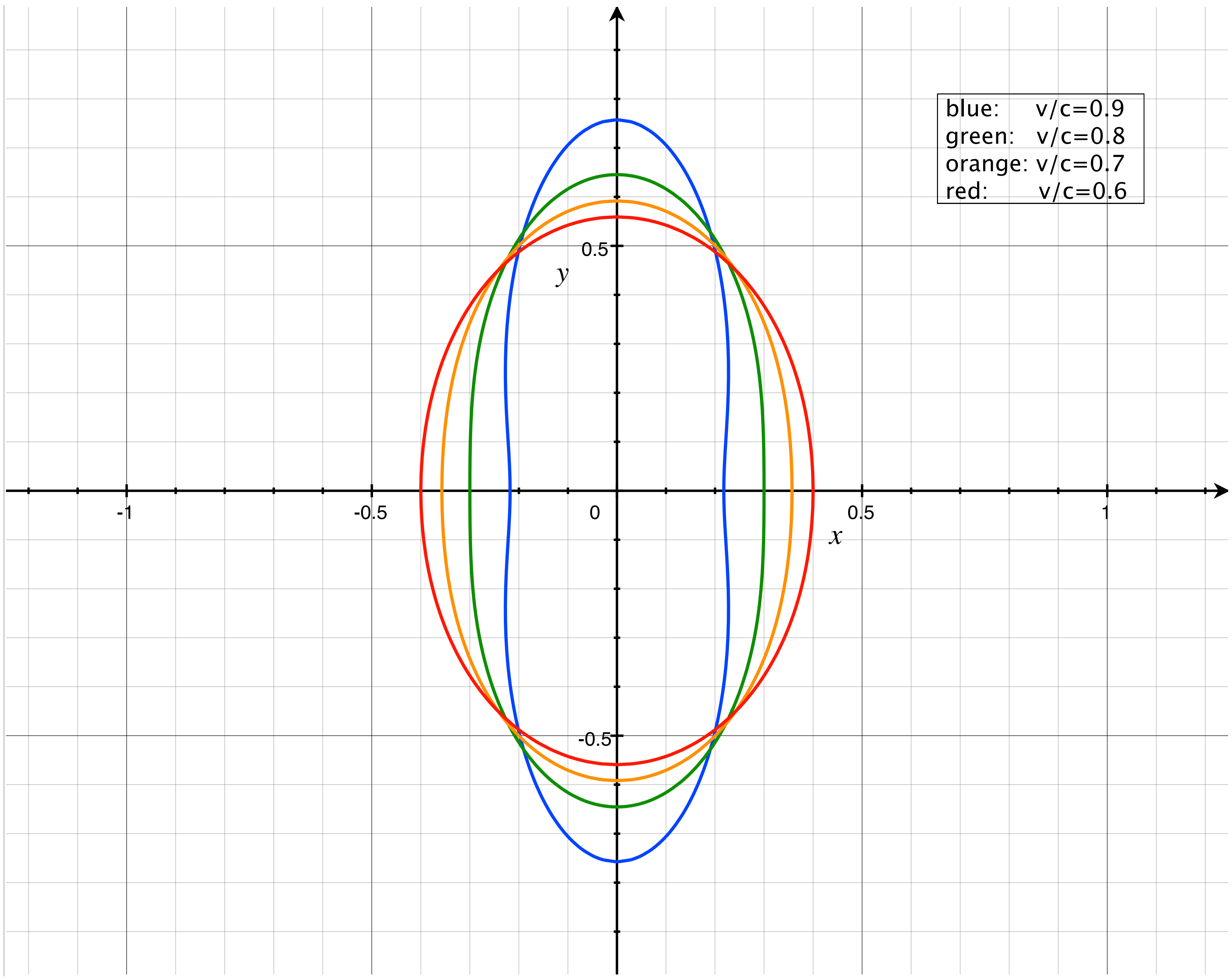
```
  }

  TCanvas *cn = new TCanvas("cn","",200,10,700,500);
  cn->Divide(2,2);

  cn->cd(1);
  gEt = new TGraph(nloop,t,E_t); gEt->SetLineColor(2); gEt->SetLineWidth(2);
  gEt->SetTitle("E field"); gEt->GetXaxis()->SetTitle("Time (s)");
  gEt->GetYaxis()->SetTitle("E field (V/m)"); gEt->Draw("AC");

  cn->cd(2);
  gEs = new TGraph(nloop,x,E_s); gEs->SetLineColor(2); gEs->SetLineWidth(2);
  gEs->SetTitle("E field"); gEs->GetXaxis()->SetTitle("Distance (m)");
  gEs->GetYaxis()->SetTitle("E field (V/m)"); gEs->Draw("AC");

  cn->cd(3);
  gBt = new TGraph(nloop,t,B_t); gBt->SetLineColor(4); gBt->SetLineWidth(2);
  gBt->SetTitle("B field"); gBt->GetXaxis()->SetTitle("Time (s)");
  gBt->GetYaxis()->SetTitle("B field (T)"); gBt->Draw("AC");

  cn->cd(4);
  gBs = new TGraph(nloop,x,B_s); gBs->SetLineColor(4); gBs->SetLineWidth(2);
  gBs->SetTitle("B field"); gBs->GetXaxis()->SetTitle("Distance (m)");
  gBs->GetYaxis()->SetTitle("B field (T)"); gBs->Draw("AC");
}
```

Electric Field of a point charge

E field of a point charge moving with constant velocity:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{1-v^2/c^2}{(1-v^2\sin^2\theta/c^2)^{3/2}}\frac{\hat{\mathbf{R}}}{R^2}$$

Eq. (10.75) Griffiths



Electric/Magnetic Dipole Radiation

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \frac{\sin^2\theta}{r^2} \hat{\mathbf{r}}$$

Eq. (11.21) Griffiths

: this is the intensity distribution as a function of 3-d space for an oscillating electric dipole. To visualize the intensity distribution, we treat the above as

$$\langle \mathrm{S} \rangle \sim \frac{\sin^2\theta}{r^2} = \frac{x^2 + y^2}{(x^2 + y^2 + z^2)^2}$$

in rectangular coordinate system.

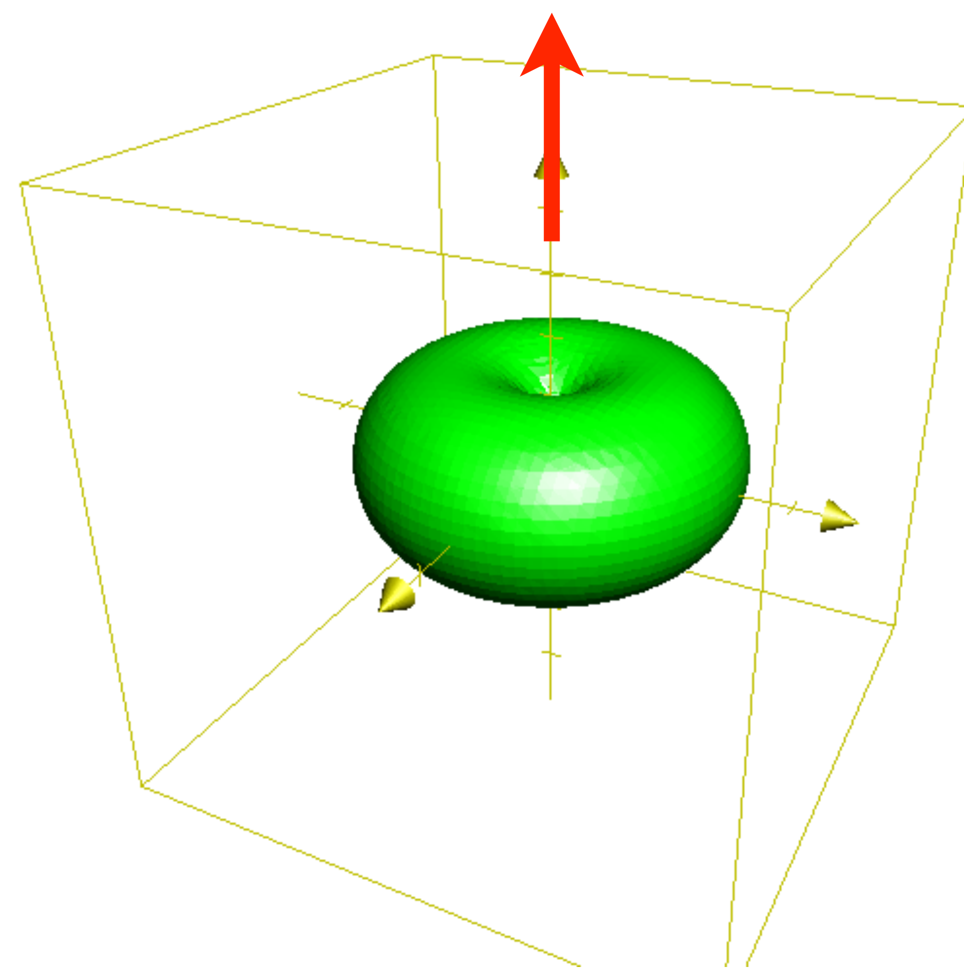
$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$x^2 + y^2 = r^2\sin^2\theta$$
$$\frac{\sin^2\theta}{r^2} = \frac{x^2 + y^2}{(x^2 + y^2 + z^2)^2}$$

And if we draw the constant surface of this function, one gets:

$$0.1 = \frac{x^2 + y^2}{(x^2 + y^2 + z^2)^2}$$

using osx Grapher.

$$\mathbf{p}(t) = p_0 \cos(\omega t)\hat{\mathbf{z}}$$



Electric/Magnetic Dipole Radiation

To see the shape closely, let's look at the 2-d slice.