6. Magnetic Fields in Matter

Unlike electric polarization, (electric polarization is almost always in the same direction as $\vec{E}$)

Some materials show magnetism

→ parallel to $\vec{B}$: paramagnet

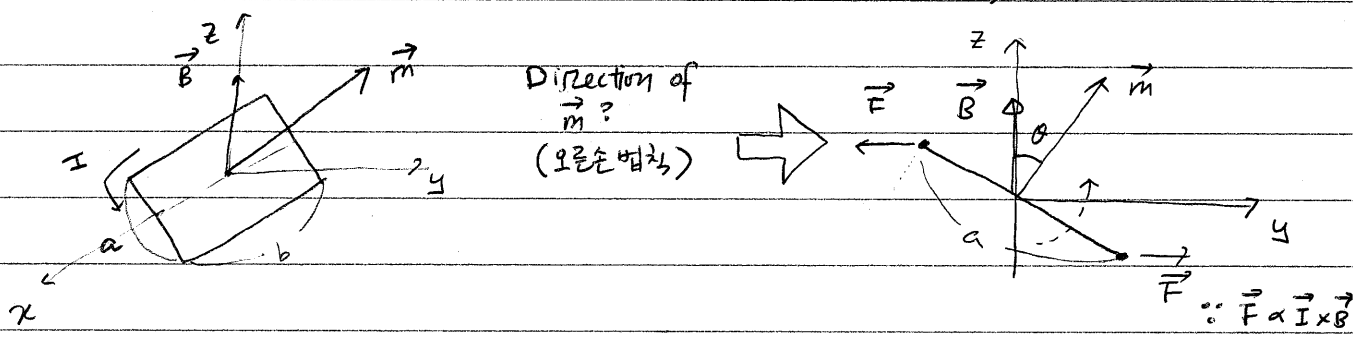
→ opposite to $\vec{B}$: diamagnet

(*) A few substances retain their magnetization after the removal of external $\vec{B}$: Ferromagnet

Torques and Forces on Magnetic Dipoles

· A magnetic dipole experiences a torque in $\vec{B}$ field.

– for a ring of current (일반물리에서 했었음)



– Torque $\vec{N}$ is formed.

$$|\vec{N}| = N = F\frac{a}{2}\sin\theta \times 2, \quad \text{direction} : +\hat{x}$$

$$\therefore \vec{N} = aF\sin\theta\,\hat{x}, \quad \text{Also we know that } F = I\,b\,B$$

length of wire piece: b

$$\therefore \vec{N} = \underbrace{a \cdot I b}_{m} B \sin\theta\,\hat{x} = \vec{m} \times \vec{B}$$

$m = I \cdot ab$ or $I \cdot$ (area of current loop)

($\vec{N} = \vec{p} \times \vec{E}$ for electric dipole)

This is the torque that accounts for paramagnetism.

(e^- constitutes a magnetic dipole → all material shows paramagnetism?

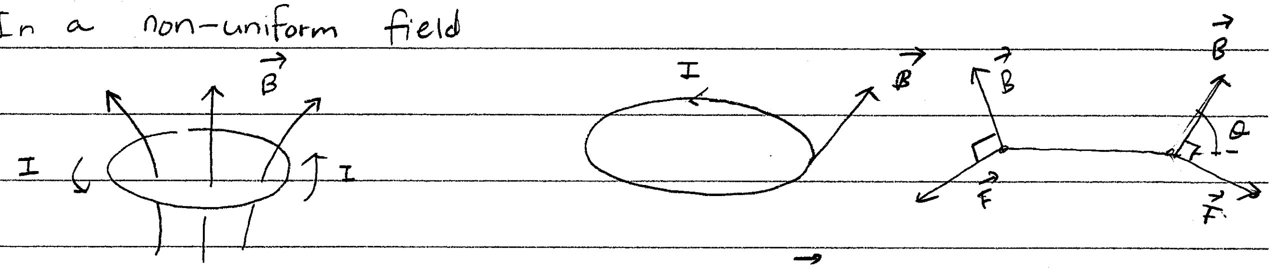
No : QM locks spin pair.)

· In a uniform field of $\vec{B}$, net force on a current loop?

$$\vec{F} = I \oint d\vec{l} \times \vec{B} = I \underbrace{\left(\oint d\vec{l} \right)}_{=0} \times \vec{B}$$

$=0$ for any closed loop

· In a non-uniform field



→ so the vertical component of $\vec{F}$ does not cancel

: net downward force

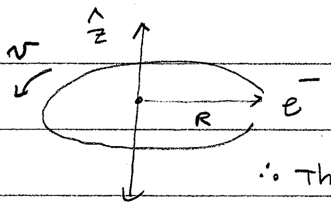
$$F = 2\pi R I B \cos\theta$$

→ For an infinitesimal loop, (w/ dipole moment $\vec{m}$)

$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})$$

(prob. 6.4, 6.22)

6.1.3 Effect of a $\vec{B}$ on Atomic Orbits



$$I = \frac{-e}{T} = -\frac{ev}{2\pi R}$$

T ← period

∴ The orbital

$\vec{m} = ?$ (= area × current)

$$= -\frac{ev}{2\pi R} \cdot \pi R^2 \hat{z} = -\frac{1}{2} e v R \hat{z}$$

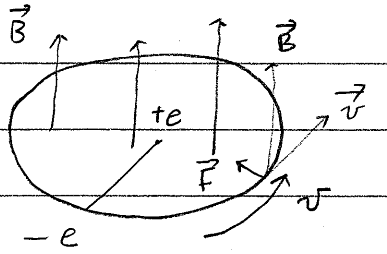
When external $\vec{B}$ field is applied, it is also subject to torque $= \vec{m} \times \vec{B}$

→ lot harder to tilt the orbit (than its spin)

→ e^- speeds up or slows down due to $\vec{B}$

For electrical force on e^- : $\frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2} = m_e \frac{v^2}{R}$

- In the presence of $\vec{B}$, there is an additional force
$: -e(\vec{v}\times\vec{B})$



So $\vec{F}$ has same sign as the Coulomb attraction force

$$\rightarrow \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B = m_e\frac{\bar{v}^2}{R}$$

$\bar{v}$: new speed of e^-

Now, $e\bar{v}B = m_e\frac{m_e}{R}(\bar{v}^2 - v^2) = \frac{m_e}{R}(\bar{v}+v)(\bar{v}-v)$

$\bar{v}$ is faster? (than v)

Above eq.s become $a = v^2$

$$a + b\bar{v} = \bar{v}^2$$

where, $a, b, v, \bar{v} > 0$. So

$\bar{v}^2 - v^2 = b\bar{v} > 0 \quad \therefore \bar{v} > v$!!

So $\bar{v}$ is greater than v.

$$e\bar{v}B = \frac{m_e}{R}(\bar{v}+v)(\bar{v}-v), \quad (\bar{v}-v) \equiv \Delta v$$

$$\Delta v \cong \frac{R}{m_e}\cdot\frac{e\bar{v}B}{2\bar{v}} = \frac{eRB}{2m_e}$$

$\therefore$ When $\vec{B}$ is on, e^- speeds up.

$\rightarrow$ Change in $\vec{m}$ is opposite to the direction of $\vec{B}$

$\rightarrow$ Responsible for diamagnetism.

6.1.4 Magnetization

The effect of matter due to external $\vec{B}$?

$\rightarrow$ State of magnetic polarization by the following vector quantity

$\vec{M}$ = magnetic dipole moment / volume

: magnetization

(analogous to $\vec{P}$: polarization)

The field of a magnetized object

- Bound current

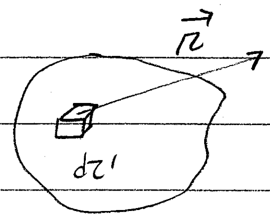
Suppose we are given with a material w/ $\vec{M}$. What field does this object produce?

Starting from the vector potential of a single dipole $\vec{m}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$\downarrow$ magnetized object has a dipole moment of $\vec{M}\,d\tau'$ on each vol. $d\tau'$

$$\therefore \quad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{M}(\vec{r}')\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau'$$



Using $\vec{\nabla}'\frac{1}{\mathscr{r}} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$, we get

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\left[\vec{M}(\vec{r}')\times\left(\vec{\nabla}'\frac{1}{\mathscr{r}}\right)\right]d\tau'$$

$\Big[$ $\vec{\nabla}\times(f\vec{V}) = f(\vec{\nabla}\times\vec{V}) - \underbrace{\vec{V}}_{\vec{M}(\vec{r}')}\times\underbrace{(\vec{\nabla}f)}_{\vec{\nabla}'\frac{1}{\mathscr{r}}}$ gives us

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left\{\int\frac{1}{\mathscr{r}}\left(\vec{\nabla}'\times\vec{M}(\vec{r}')\right)d\tau' - \int\vec{\nabla}'\times\left(\frac{\vec{M}(\vec{r}')}{\mathscr{r}}\right)d\tau'\right\}$$

is obtained. $\Big]$ Now from

$$\int_{\mathcal{V}}\nabla\times\vec{v}\,d\tau = -\oint_{S}\vec{v}\times d\vec{a}$$

we get

$$\vec{A}(\vec{r}) = \underbrace{\frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}\left[\vec{\nabla}'\times\vec{M}(\vec{r}')\right]d\tau'}_{\text{: Volume integral}} + \underbrace{\frac{\mu_0}{4\pi}\oint\frac{\vec{M}(\vec{r}')}{\mathscr{r}}\times d\vec{a}}_{\text{: surface integral}}$$

↑ potential of a volume current ↑ potential of a surface current

So, if we define

$$\underbrace{\vec{J}_b}_{\text{bound}} = \vec{\nabla}\times\vec{M} \quad \text{and} \quad \vec{K}_b = \vec{M}\times\underbrace{\hat{n}}_{\text{normal unit vector}}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_v \frac{\vec{J}_b(\vec{r}')}{\mathscr{r}}\,d\tau' + \frac{\mu_0}{4\pi}\oint_s \frac{\vec{K}_b(\vec{r}')}{\mathscr{r}}\,da'$$

potential of magnetised object = Volume current ($\vec{J}_b = \vec{\nabla}\times\vec{M}$) + surface current ($\vec{K}_b = \vec{M}\times\hat{n}$)

, instead of integrating the contributions of all the infinitesimal dipoles!

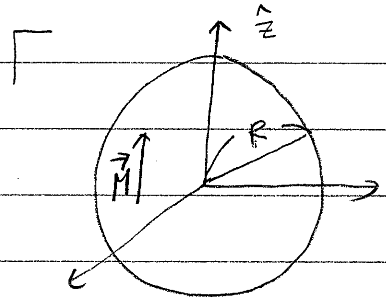
$\Leftarrow$ "bound current"

Note : in $\vec{E}$, we had "bound charge"

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$\sigma_b = \vec{P}\cdot\hat{n}$$

6.1 Ex) $\vec{B}$ of uniformly magnetized sphere?



$$\vec{J}_b = \vec{\nabla}\times\vec{M} = \vec{0}$$

$$\vec{K}_b = \vec{M}\times\hat{n} = ?$$

$$\vec{M} = M\hat{z} = M(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

$$\hat{n} = \hat{r}$$

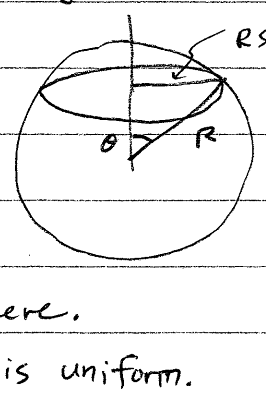
$$\therefore\ \vec{M}\times\hat{n} = M(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})\times\hat{r} = -M\sin\theta\,\hat{r}\times\hat{\theta} = M\sin\theta\,\hat{\phi}$$

$$\therefore\ \vec{K}_b = M\sin\theta\,\hat{\phi}$$

- In Ex 5.11) (Rotating shell of surface charge density)

$$\vec{B} = \frac{2}{3}\mu_0\sigma R\vec{\omega}$$

- For a rotating spherical shell,



$$\vec{K} = \sigma\vec{v} = \sigma\omega R\sin\theta\,\hat{\phi} =$$

$M = \sigma\omega R$ or $\vec{M} = \sigma\vec{\omega}R$

$\therefore\ \vec{B} = \frac{2}{3}\mu_0\vec{M}$ inside the sphere.

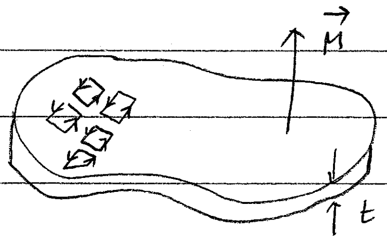
$\therefore$ internal field is uniform.

Outside? $\vec{M} = \vec{m}/\text{volume}$ ($\vec{m}$: magnetic dipole moment)

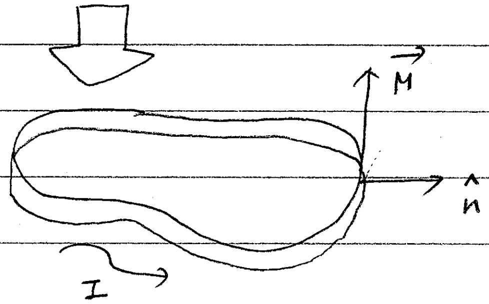
$\rightarrow\ \vec{M} = \vec{m}/\frac{4}{3}\pi R^3 \ \rightarrow\ \vec{m} = \frac{4\pi}{3}R^3\vec{M}$

6.2.2 Physical Interpretation of Bound Currents

- Suppose a thin slab (II_t) of uniformly magnetized material with dipoles (tiny current loops)



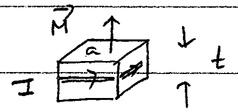
- internal currents cancel
- Edge : no cancelation.



- a sigle ribbon of current I around the boundary!

- I and $\vec{M}$?

For each tiny loop (∵ $M = m/\text{volume}$)



dipole moment $m = Mat$

or $m = Ia$ (∵ m = current × area)

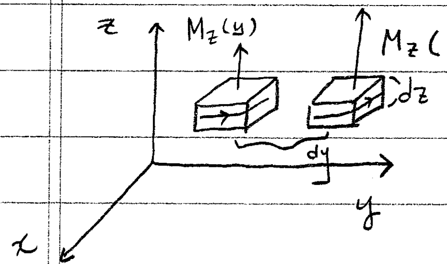
$\rightarrow \quad I = Mt$

$\rightarrow \quad K_b = I/t = M \quad$ or $\quad \vec{K_b} = \vec{M} \times \hat{n}$

(so direction of $\vec{K_b}$ is direction of current.)

- If $\vec{M}$ is not uniform :

↳ the internal currents no longer cancel.



On the surface where they join ∃ net current in $+x$ direction

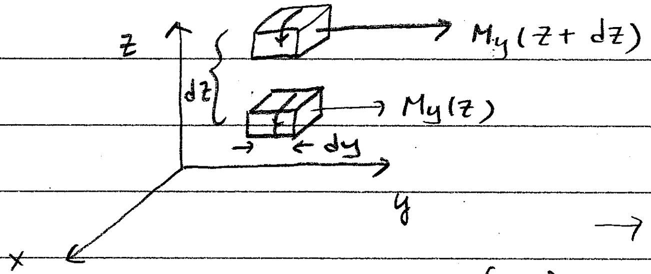
$$\therefore I_x = \left[M_z(y+dy) - M_z(y) \right] dz \qquad (\because I = Mt)$$

$$= \frac{\partial M_z}{\partial y} dy\, dz$$

Since $I_x = \frac{\partial M_z}{\partial y} dy\, dz$ (and $I = J \cdot \text{area}$)

$$(J_b)_x = \frac{\partial M_z}{\partial y}.$$

Now, for a nonuniform magnetization in the y direction



This time, the current is (−) x direction.

$$\rightarrow (J_b)_x = -\frac{\partial M_y}{\partial z}$$

So we have $(J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z} = (\nabla \times \vec{M})_x$

→ In general, $\vec{J}_b = \vec{\nabla} \times \vec{M}$ can be shown

so, automatically $\vec{\nabla} \cdot \vec{J}_b = 0$ is satisfied.

6.2.3 The Magnetic Field Inside Matter

microscopic $\vec{B}$: too complicated to deal with

→ we are interested in the macroscopic field.

(보조장)

6.3 The auxiliary field $\vec{H}$

- We learned that $\vec{M}$ causes bound current density
  bound current $\vec{J}_b = \vec{\nabla} \times \vec{M}$ and $\vec{K}_b = \vec{M} \times \hat{n}$
- There is also field not due to $\vec{M}$
  ↳ we call free current for the field not due to $\vec{M}$

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

"Ampere's law"

From $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, we have

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = \vec{J} = \vec{J}_b + \vec{J}_f = \vec{J}_f + \vec{\nabla} \times \vec{M}$$

or, $$\vec{\nabla} \times \underbrace{\left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right)}_{\equiv \vec{H}} = \vec{J}_f$$

so $\vec{\nabla} \times \vec{H} = \vec{J}_f$

or $\oint \vec{H} \cdot d\vec{l} = I_{f_{enc}}$

$\vec{H}$ allows us to express Ampère's law in terms of free current alone.

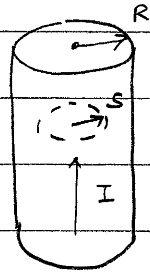
Q: $\vec{M}$ not given. Is it OK?

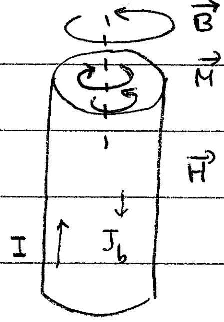
Ex 6.2) A long copper rod of radius R, w/ uniformly distributed (free) current I, $\vec{H}$ inside and outside the rod.

Copper : weakly diamagnetic $\rightarrow$ dipoles line up opposite to the field.

$\rightarrow$ $\exists$ bound current antiparallel to I.

Amperian loop





$\vec{M}$: $\vec{M}$ is opposite to $\vec{B}$ since copper is diamagnetic

$\vec{H}$: same direction as $\vec{B}$ since it should be from $\vec{J}_f$ and $|\vec{M}| \ll 1$.

i) $s < R$

same as $\hat{B}$

$\vec{\nabla} \times \vec{H} = \vec{J}_f$ gives us (or $\oint \vec{H} \cdot d\vec{l} = I_{fenc}$)

$$H \cdot 2\pi s = I_{fenc} = I \frac{\pi s^2}{\pi R^2}, \quad \vec{H} = \frac{I}{2\pi R^2} s \,\hat{\phi}$$

outside ii) $s \geq R$

$$\vec{H} = \frac{I}{2\pi s}\hat{\phi} \quad \text{and} \quad \vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

($\vec{M} = 0$ at $s > R$)

- Boundary conditions

$$\left.\begin{array}{l} B^{\perp}_{above} = B^{\perp}_{below} \\ \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \end{array}\right\} \text{ gives us } \quad H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$$

$$\oint \vec{H} \cdot d\vec{l} = I_{fenc} \longrightarrow \vec{H}^{\parallel}_{above} - \vec{H}^{\parallel}_{below} = \vec{K} \times \hat{n}$$

(see eq. 5.75 / 5.76)

- Linear / Nonlinear Media

민감성

  - Magnetic Susceptibility and permeability

We had

$\vec{P} = \epsilon_0 \chi_e \vec{E}$. So we might want to define

$\vec{M} = \frac{1}{\mu_0} \chi_m \vec{B}$? ⊗

No, the real definition is (unfortunately?!)

$$\vec{M} = \chi_m \vec{H}$$

χ_m : ✓ magnetic susceptibility

✓ dimensionless, material specific

✓ signed quantity

$\chi_m > 0$: paramagnets

$\chi_m < 0$: diamagnets

Materials obeying it are called <u>linear media</u>

or $\vec{B} = \mu_0 (\vec{M} + \vec{H}) = \mu_0 (1 + \chi_m) \vec{H}$

$\equiv \mu \vec{H}$, $\mu \equiv \mu_0 (1 + \chi_m)$

permeability of the material

- Ferromagnets (강자성체 / 자석)

: requires quantum mechanics to understand better