0 Preliminary

In your general physics class:

we covered Maxwell's eqns completely

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$ Gauss's law for electricity

closed; S ← surface integral

ϵ_0 ← $8.85\times10^{-12}\ C^2/N\cdot m^2$ (permittivity constant)

$$\oint_S \vec{B}\cdot d\vec{a} = 0$$ Gauss's law for magnetism

$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ Faraday's law

Φ_B ← Magnetic flux

closed; Γ ← Line integral

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$$ Ampere-Maxwell law

Φ_E ← electric flux

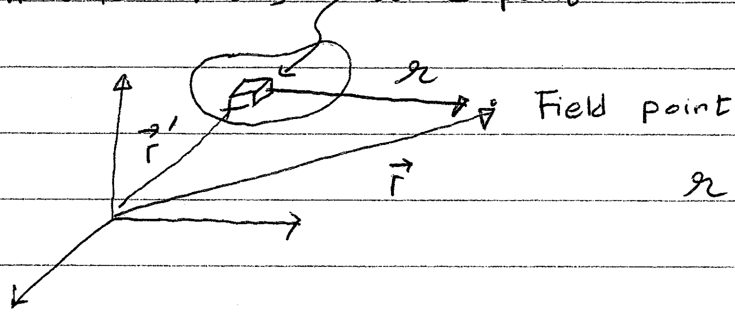
μ_0 ← $4\pi\times10^{-7}\ T\cdot m/A$ (permeability constant)

$\mu_0\epsilon_0 = \frac{1}{c^2}$, c ← speed of light.

1. Vector Analysis

: this is the content you all are (or must be) familiar with!

In this book,



$$\mathscr{r} \equiv \vec{r} - \vec{r}\,'$$

Vector differential calculus & vector integral calculus

- We learned

$\vec{\nabla}$: gradient
$\vec{\nabla}\cdot$: divergence
$\vec{\nabla}\times$: curl

} operators in mathematical physics

Do you understand meaning of these operators?

Ex) Divergence operator

① $\vec{v} = r\hat{r}$, $\vec{\nabla}\cdot\vec{v} = ?$ $(=0?)$ $\neq 3$

② $\vec{v} = \hat{z}$ $\vec{\nabla}\cdot\vec{v} = ?$

③ $\vec{v} = z\hat{z}$ $\vec{\nabla}\cdot\vec{v} = ?$

Can you make sense out of two?

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ but $\vec{\nabla}\cdot\vec{E} = ?$
at $r=0$, $\vec{E}$ cannot be defined

$\vec{\nabla}\cdot\vec{v}$: it is a measure of how much the vector $\vec{v}$ spreads out (diverges) from a point in question

- what about curl? (see text p19)

- Fundamental Theorem for Divergence
  - For a given vector field $\vec{v}$

$$\int_V \vec{\nabla}\cdot\vec{v}\, d\tau = \oint_S \vec{v}\cdot d\vec{a}$$

V: volume integral; $\oint_S$: closed surface

$\int_V \vec{\nabla}\cdot\vec{E}\, d\tau = \frac{1}{\epsilon_0}\int \rho\, d\tau$

Taking $\oint_S \vec{E}\cdot d\vec{a} = \frac{I_{enc}}{\epsilon_0}$

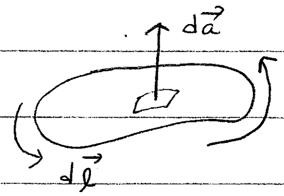
⇓

differential exp. of Gauss's law → $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

- Fundamental Theorem for Curls

$$\int_S (\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \oint_P \vec{v}\cdot d\vec{l}$$

P: path (line)



$\int(\vec{\nabla}\times\vec{E})\cdot d\vec{a}$
$\parallel$
$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$

$\therefore \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$: Faraday's law (diff. form)

Use right hand!

- problem 1.63, 1.61
- Eq. 1.100 in P 232
- Review of GP 101 EM

(HW) 1.13 1.16 1.38 1.39, 1.61, 1.62 1.63

Dirac said: "I understand what an equation means if I have a way of figuring out the characteristics of its solution without actually solving it"

↓

A physical understanding is a completely unmathematical, imprecise, and inexact thing, but absolutely necessary for a physicist.