http://particle.korea.ac.kr/class/2005/phys222/

1. $\vec{c} \times \vec{c} = 0$

$(\vec{A} + \vec{B}) \times (\vec{A} \times \vec{B}) = 0$

$\vec{A} \times \vec{A} + \vec{A} \times \vec{B} + \vec{B} \times \vec{A} + \vec{B} \times \vec{B} = \vec{A} \times \vec{B} + \vec{B} \times \vec{A} = 0$

2. $\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}$

$A_x = \frac{m_y x}{r^3} - \frac{m_x r_y}{r^3} \qquad A_y = \frac{m_z x}{r^3} - \frac{m_x z}{r^3} \qquad A_z = \frac{m_x y}{r^3} - \frac{m_y r x}{}$

$$(\vec{\nabla} \times \vec{A})_x \cdot \frac{4\pi}{\mu_0} = \frac{\partial}{\partial y}\underbrace{\left[\frac{m_x y}{r^3} - \frac{m_y x}{r^3}\right]}_{A_z} - \frac{\partial}{\partial z}\underbrace{\left[\frac{m_z x}{r^3} - \frac{m_x z}{r^3}\right]}_{A_y}$$

Note: $r = \sqrt{x^2 + y^2 + z^2}$, $\frac{\partial r}{\partial x} = \frac{x}{r}$

$$\therefore (\vec{\nabla} \times \vec{A})_x \cdot \frac{4\pi}{\mu_0} = \left[\frac{m_x}{r^3} - 3\frac{m_x y}{r^4} \cdot \frac{y}{r} + 3\frac{m_y x}{r^4} \frac{y}{r}\right]$$

$$\left[-3\frac{m_z x}{r^4} \cdot \frac{z}{r} - \frac{m_x}{r^3} + 3\frac{m_x z}{r^4} \cdot \frac{z}{r}\right]$$

$$= \frac{m_x}{r^3} - 3\frac{m_x y^2}{r^5} + 3\frac{m_y xy}{r^5} + \frac{3 m_z xz}{r^5} + \frac{m_x}{r^3} - 3\frac{m_x z^2}{r^5}$$

$$= \frac{1}{r^5}\left[m_x r^2 - 3m_x y^2 + 3m_y xy + 3m_z xz + m_x r^2 - 3m_x z^2\right]$$

$$= \frac{1}{r^5}\left[2m_x r^2 - 3m_x y^2 + 3m_y xy + 3m_z xz - 3m_x z^2\right]$$

$$= \frac{1}{r^5}\left[3m_x r^2 - m_x r^2 - 3m_x y^2 + 3m_y xy + 3m_z xz - 3m_x z^2\right]$$

$$= \frac{1}{r^5}\left[3m_x (x^2 + y^2 + z^2) - m_x r^2 - 3m_x y^2 + 3m_y xy + 3m_z xz - 3m_x z^2\right]$$

$$= \frac{1}{r^5}\left[3x(x m_x + y m_y + z m_z) - m_x r^2\right]$$

Now, $\vec{B} = \frac{\mu_0}{4\pi} \frac{3\hat{r}(\hat{r} \cdot \vec{m}) - \vec{m}}{r^3} = \frac{\mu_0}{4\pi} \frac{3\vec{r}(\vec{r} \cdot \vec{m}) - \vec{m} r^2}{r^5}$

X component of $\vec{B}$: $B_x = \frac{\mu_0}{4\pi} \frac{3x(x m_x + y m_y + z m_z) - m_x r^2}{r^5}$

$\therefore (\vec{\nabla} \times \vec{A})_x = \vec{B}_x$. Repeating this for y and z component we get $\vec{\nabla} \times \vec{A} = \vec{B}$

3. $\vec{F} = \vec{\nabla} U$

U is a function of R only. So $\vec{F} = \frac{dU}{dR}\frac{\vec{R}}{R}$, $\vec{R} = x$

For $R \le a$,

$$\frac{dU}{dR} = -\pi b R \quad , \quad \vec{F} = -\pi b \, (x\hat{i} + y\hat{j})$$

For $R > a$

$$\frac{dU}{dR} = -\pi b a^2 \frac{1}{R} \quad \vec{F} = -\pi b a^2 \frac{1}{R^2} (x\hat{i} + y\hat{j})$$

4. we know that

$$\int_S \vec{v} \cdot d\vec{\sigma} = \int \vec{\nabla} \cdot \vec{v} \, d\tau$$

Assume $\vec{v}$ is a constant vector field:

Then $\vec{v} \cdot \int d\vec{\sigma} = \int_V \vec{\nabla} \cdot \vec{v} \, d\tau = 0$ $\quad (\because \vec{v}$ is constant$)$

$$\therefore \int d\vec{\sigma} = 0$$

5. $S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$

$$= \frac{1}{2^n}(2^{n-1} + 2^{n-2} + \cdots + 4 + 2 + 1)$$

Note that $(1 + 2 + 4 + \cdots + 2^{n-1})$

$$= (2-1)(1 + 2 + 4 + \cdots + 2^{n-1})$$

$$= 2 + 4 + 8 + \cdots + 2^{n-1} + 2^n$$
$$- 1 - 2 - 4 - 8 - \cdots - 2^{n-1}$$

$$= 2^{n-1}$$

$$\therefore S_n = \frac{1}{2^n}(2^n - 1), \quad \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{2^n - 1}{2^n} = 1$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

6. $\int_2^{\infty} \frac{1}{x \log x} dx = \log(\log x) \Big|_2^{\infty} = \infty$ $\quad \therefore \sum_{n=2}^{\infty} \frac{1}{n \log n}$ dive

7 $\int_S (\vec{v} \times \vec{A}) \cdot d\vec{\sigma} = \int \vec{\nabla} \cdot (\vec{v} \times \vec{A})\, d\tau$

$$\vec{\nabla} \cdot (\vec{v} \times \vec{A}) = \vec{A} \cdot (\vec{\nabla} \times \vec{v}) - \vec{v} \cdot \underbrace{(\vec{\nabla} \times \vec{A})}_{=0}$$

$$\therefore \int_S (\vec{v} \times \vec{A}) \cdot d\vec{\sigma} = \int \vec{A} \cdot (\vec{\nabla} \times \vec{v})\, d\tau$$

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$$\begin{pmatrix} 1 & -1 & 0 \\ -1 & \frac{11}{7} & -\frac{1}{7} \\ 0 & -\frac{1}{7} & \frac{2}{7} \end{pmatrix}$$

9 $\int_{-\infty}^{\infty} \delta'(x) f(x) dx = \delta(x) f(x) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(x) f'(x) dx$

$$= f'(0)$$

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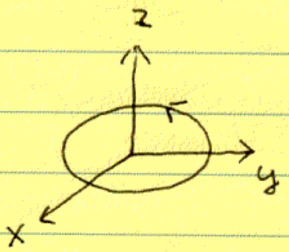

normal vector : $+\hat{z}$