PHY222 (Mathematical Physics I: Correction)

December 2nd, 2005

I was wrong two times. Here is the correction. We derived the below formula from the indefinite integral,

$$\psi(z+1) = C_1 + \ln z + \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{(2n)z^{2n}}.$$

Note that there is **NO** other term containing unity. Now, by integrating above from $z-1$ to z, we get

$$\begin{aligned} C_1 &= \lim_{z\to\infty} \ln\left(\frac{(z-1)^{z-\frac{1}{2}}}{z^{z-\frac{1}{2}}}\right) + 1 \\ &= \lim_{z\to\infty} \ln\left((1-\frac{1}{z})^{z-\frac{1}{2}}\right) + 1 \end{aligned}$$

Since we know that

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$

C_1 becomes

$$\begin{aligned} C_1 &= \lim_{z\to\infty} \ln\left((1-\frac{1}{z})^{z-\frac{1}{2}}\right) + 1 \\ &= \ln e^{-1} + 1 \\ &= 0. \end{aligned}$$

Therefore, the digamma function becomes

$$\psi(z+1) = \ln z + \frac{1}{2z} - \sum_{n=1}^{\infty} \frac{B_{2n}}{(2n)z^{2n}},$$

and now everything is in good agreement.