PHYS151 Lecture 15

Ch 15 Fluids

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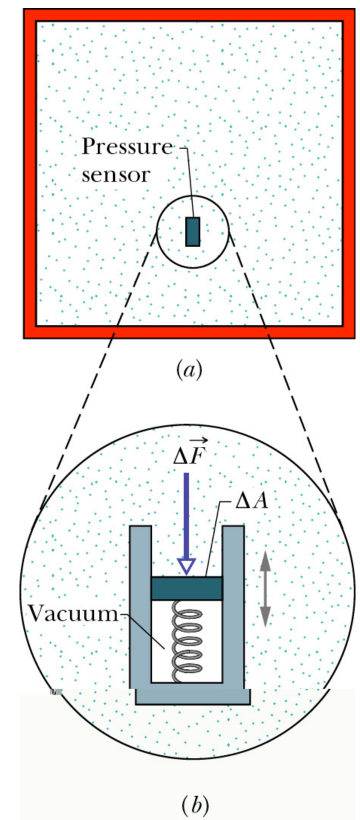
Density and Pressure

What is a fluid (유체)?
a substance that can flow (liquids and gases)

Density ρ of a fluid at any point $\rho = \dfrac{\Delta m}{\Delta V}$ or $\rho = \dfrac{m}{V}$ if it is uniform

SI unit of density : kg/m^3

Pressure on the piston from the fluid (see the figure)



A fluid-filled vessel containing a small pressure sensor

ΔA : area of a piston
ΔF : the force that acts normal to the piston

$$p = \frac{\Delta F}{\Delta A}$$

if the force is uniform,

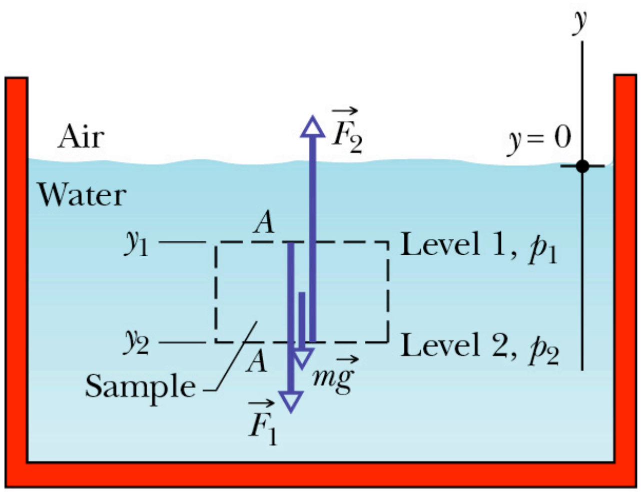
$$p = \frac{F}{A}$$

SI unit : pascal (Pa) = N/m^2

1 atm = 1.1×10^5 Pa
(atm is the approximate average pressure of the atmosphere at sea level)

Fluids at Rest

Let's imagine a right circular cylinder of horizontal base



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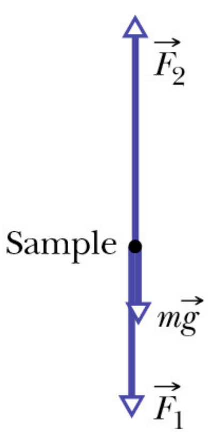


(b)

free body diagram

A : base area of the cylinder

y_1,y_2: depths below the surface of the upper and lower cylinder faces

F_1: force acts at the top surface of the cylinder

F_2: force acts at the bottom surface

The water is in static equilibrium $$F_2 = F_1 + mg$$

$$F_1 = p_1 A \quad \text{and} \quad F_2 = p_2 A$$

the mass m of the water in the cylinder is: $$m = \rho A(y_1 - y_2)$$

$$p_2 A = p_1 A + \rho A g(y_1 - y_2)$$

or $$p_2 = p_1 + \rho g(y_1 - y_2)$$

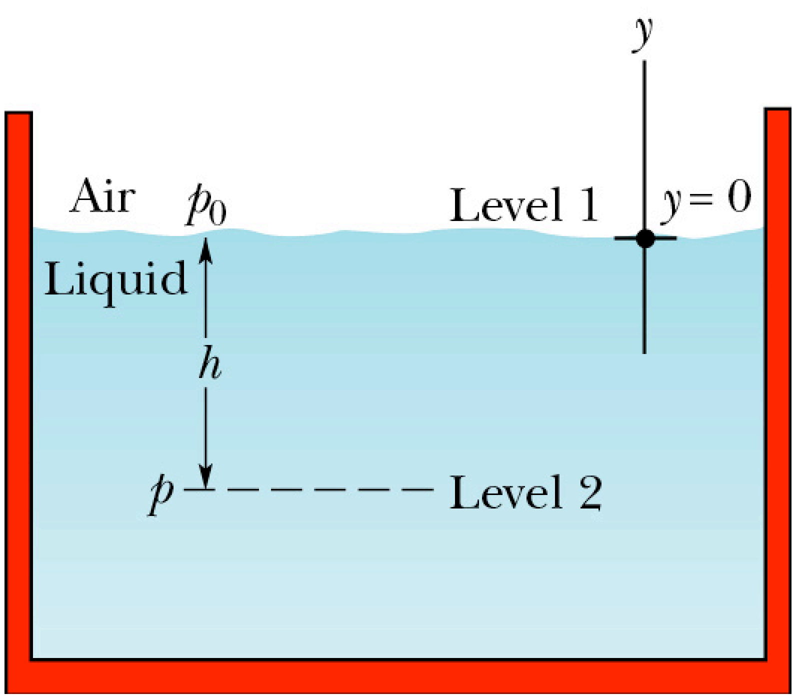
$$y_1 = 0, p_1 = p_0 \quad \text{and} \quad y_2 = -h, p_2 = p$$

$$p = p_0 + \rho g h$$

Fluids at Rest

The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container

$$p = p_0 + \rho g h$$

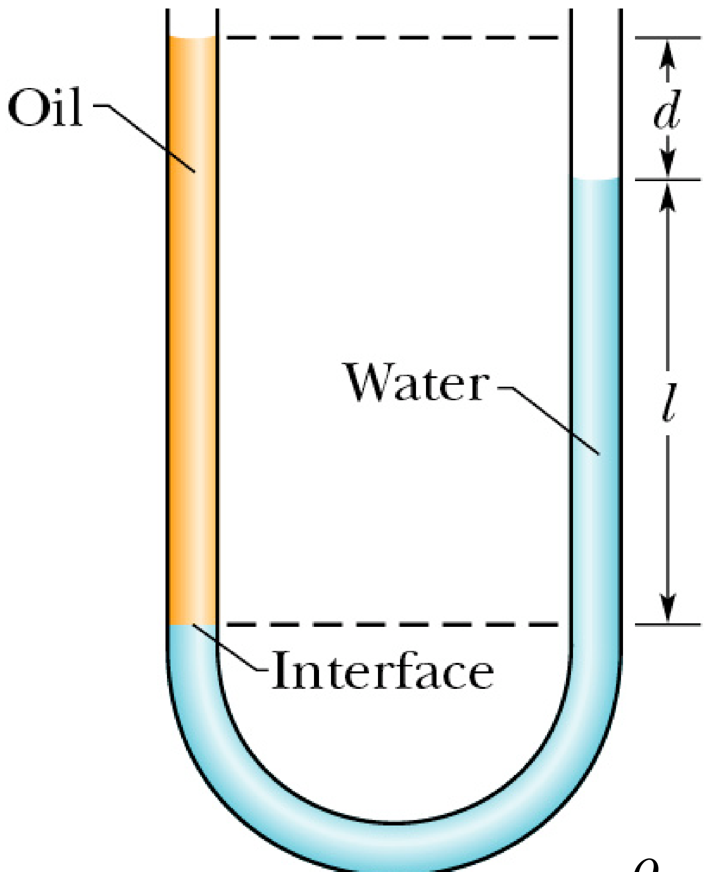


The pressure p in the above equation is said to be the total pressure or absolute pressure

The difference between an absolute pressure and an atmospheric pressure is called gauge pressure

Fluids at Rest

Sample Problem



The U-tube contains two liquids in static equilibrium: Water of density ρ_w (=998 kg/m^3) is in the right arm, and oil of unknown density ρ_x is in the left.
Measurement gives l = 135 mm and d = 12.3 mm.
What is the density of the oil?

right arm:

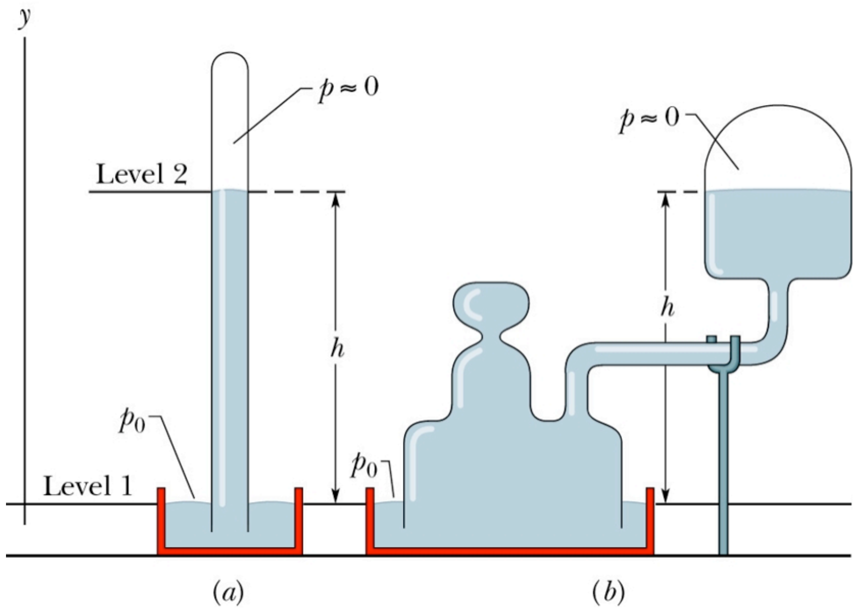
$$p_{int} = p_0 + \rho_w g l$$

left arm:

$$p_{int} = p_0 + \rho_x g(l + d)$$

$$\begin{aligned} \rho_x &= \rho_w \frac{l}{l+d} = (998\mathrm{kg/m^3}) \frac{135\ mm}{135\ mm + 12.3\ mm} \\ &= 915\mathrm{kg/m^3} \end{aligned}$$

Measuring Pressure

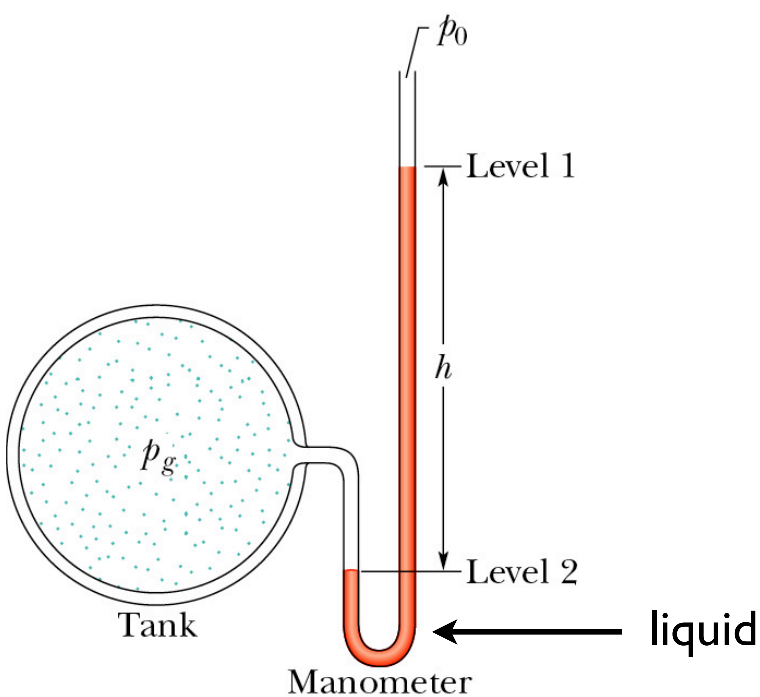


Mercury barometer: a device used to measure the pressure of the atmosphere

density of mercury: 13.5462 g/cm^3 @ 20 $^{\circ}$C

$$y_1 = 0, p_1 = p_0 \quad \text{and} \quad y_2 = h, p_2 = 0$$

$$p_0 = \rho g h$$



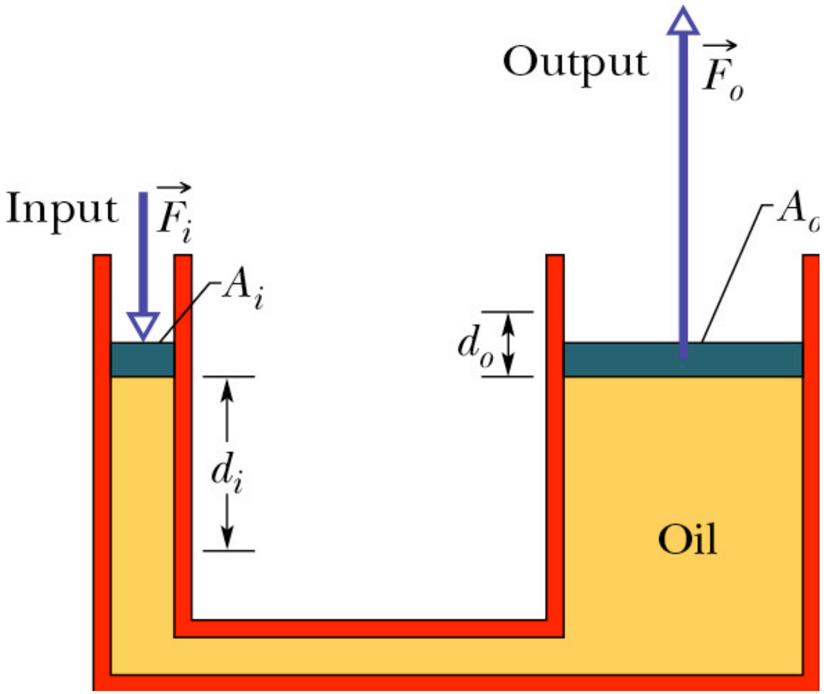
Open-Tube Manometer: measures the gauge pressure of a gas

$$y_1 = 0, p_1 = p_0 \quad \text{and} \quad y_2 = -h, p_2 = p$$

$$p_g = p - p_0 = \rho g h$$

Pascal's Principle (and the Hydraulic Lever)

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container



F_i : directed downward on the left hand piston

A_i: area of the left hand piston

F_o: upward force on the right hand piston

A_o: area of the right hand piston

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o} \quad \text{so} \quad F_o = F_i \frac{A_o}{A_i}$$

output force on the load must be greater than the input force if $A_o > A_i$

If we move the input piston downward a distance d_i, the output piston moves upward a distance d_o (same volume of the liquid is displaced)

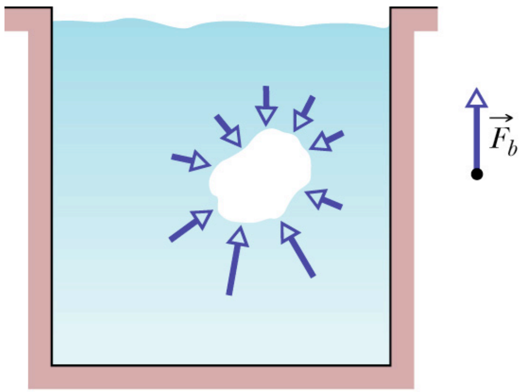
$$V = A_i d_i = A_o d_o \quad \text{so} \quad d_o = d_i \frac{A_i}{A_o}$$

output piston moves a smaller distance if $A_o > A_i$

output work can be written as

$$W = F_o d_o = \left(F_i \frac{A_o}{A_i}\right)\left(d_i \frac{A_i}{A_o}\right) = F_i d_i$$

Archimedes' Principle



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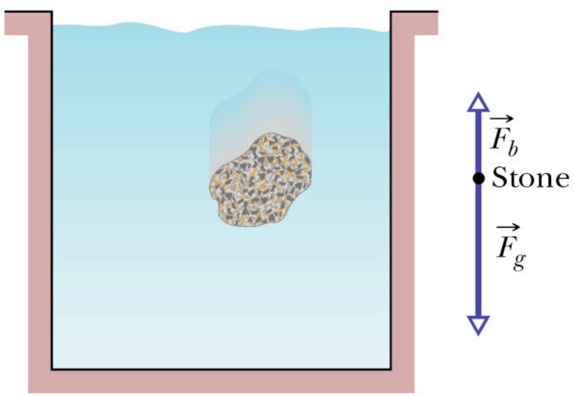

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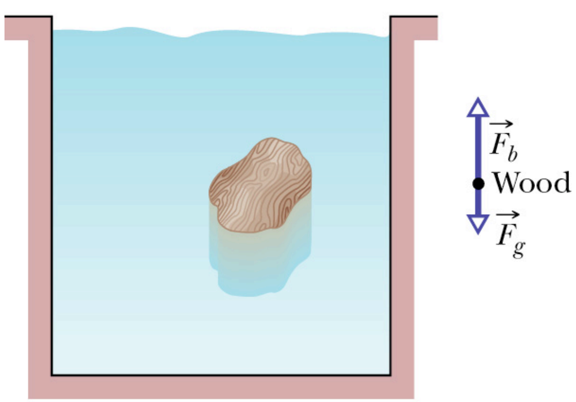


(c)

When a body is in a fluid, there is a net upward force called buoyant force

The buoyant force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body

$$F_b = m_f g$$

Floating

When a body floats in a fluid, the magnitude of the buoyant force is equal to the magnitude of the gravitational force

$$F_b = F_g$$

in other words, a floating body displaces its own weight of fluid

Apparent Weight in a Fluid

in a fluid:

$$\left(\begin{array}{c}\text{apparent}\\ \text{weight}\end{array}\right) = \left(\begin{array}{c}\text{actual}\\ \text{weight}\end{array}\right) - \left(\begin{array}{c}\text{magnitude of}\\ \text{buoyant force}\end{array}\right)$$

(easy to lift a heavy stone underwater)

Ideal Fluids in Motion

The motion of an ideal fluid

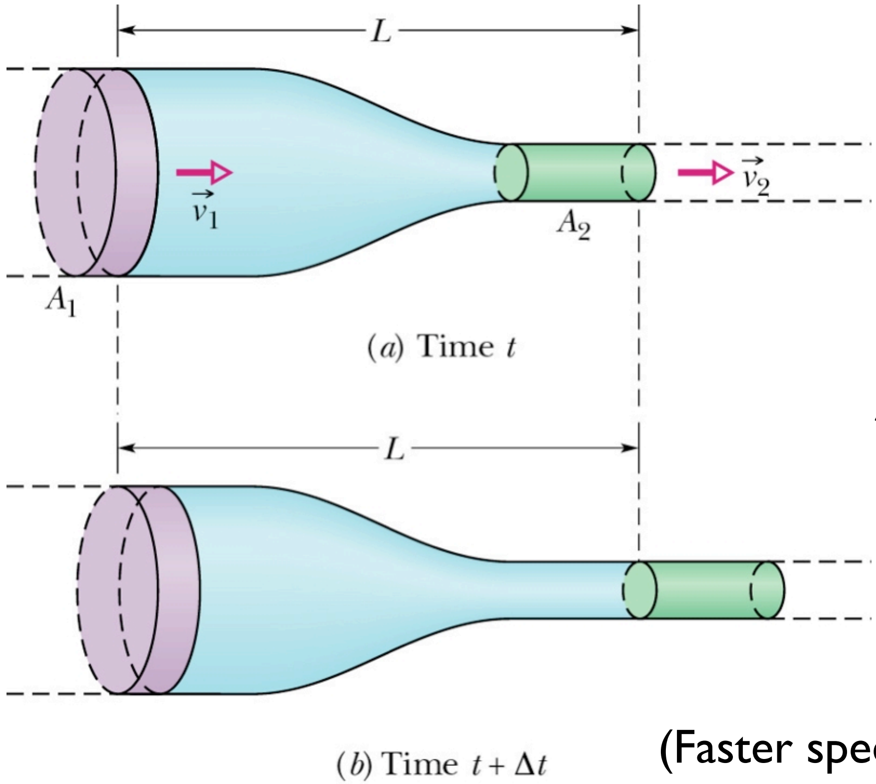
1. Steady flow: the velocity of the moving fluid at any fixed point does not change with time
2. Incompressible flow: the density of the fluid is constant, uniform value
3. Nonviscous flow: no resistive force due to viscosity

   Note: honey is said to be more viscous than water.
   viscosity is the fluid analog of friction between solids

4. Irrotational flow: no rotational flow

   In irrotational flow a test body will not rotate about an axis through its own center of mass

The Equation of Continuity



Fluid flows from left to right at a steady rate through a tube segment of length L

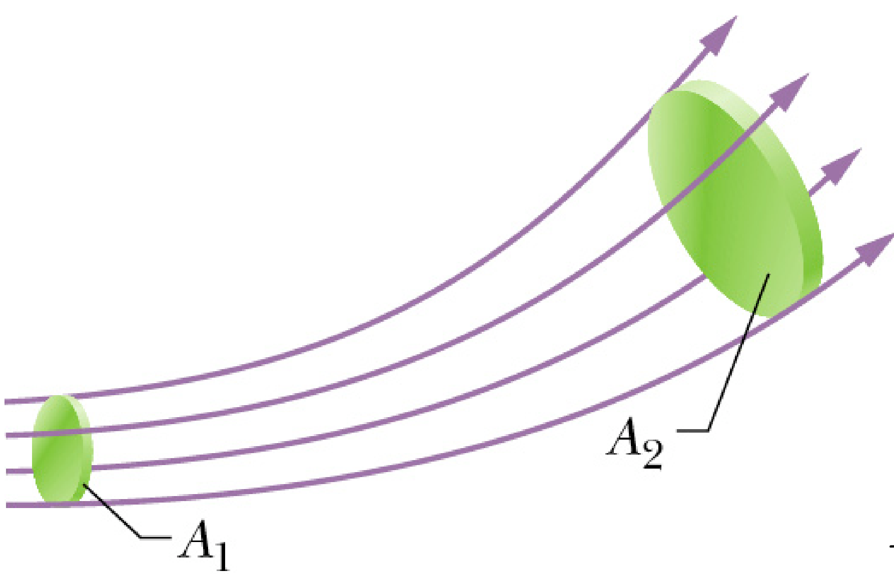
Since the fluid is incompressible

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

so $$A_1 v_1 = A_2 v_2$$

(equation of continutity)

(Faster speed with partially closing off a garden hose with a thumb)



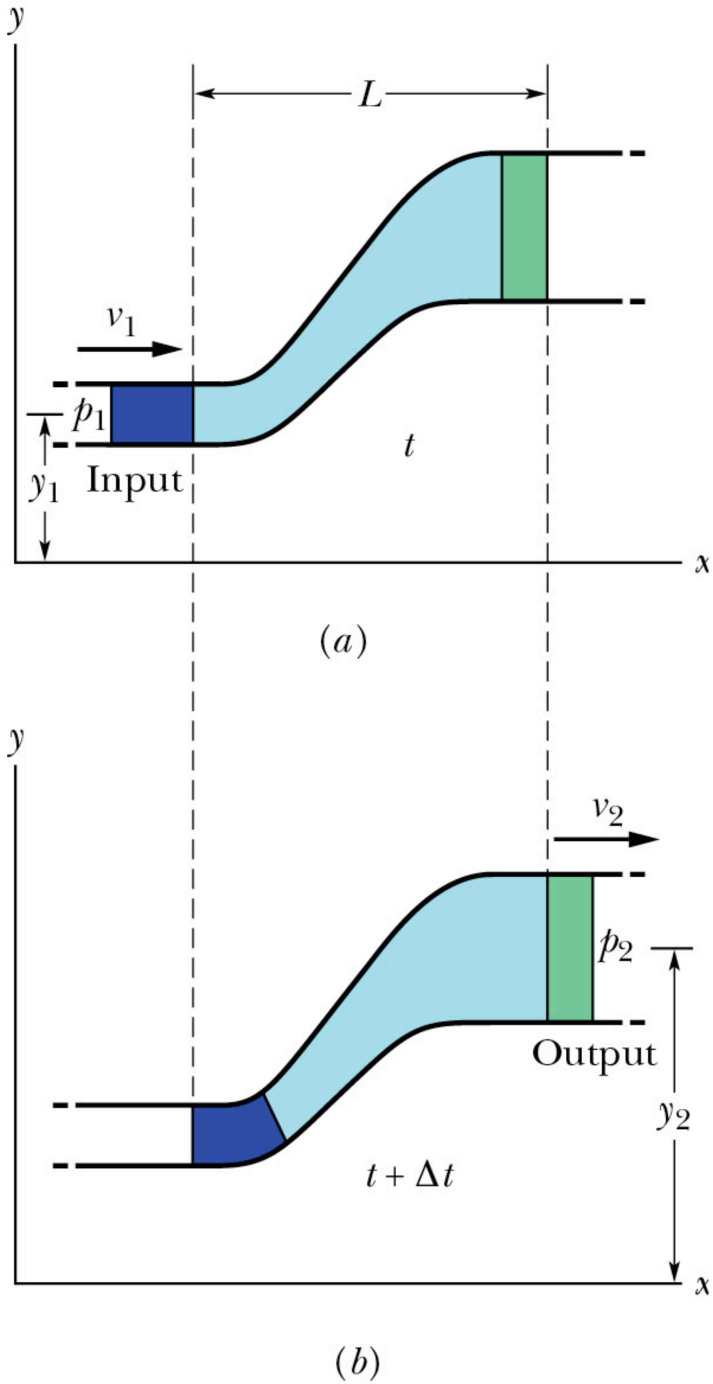
volume flow rate (SI unit: m^3/s)

$$R_V = Av = \text{a constant}$$

mass flow rate (SI unit: kg/s)

$$R_m = \rho R_V = \rho A v = \text{a constant}$$

Bernoulli's Equation



An ideal fluid is flowing at a steady rate from left to right

applying the conservation of energy to the fluid:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

or $$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant}$$

(Bernoulli's equation)

Fluids at rest ($v_1 = v_2 = 0$), then Bernoulli's equation becomes

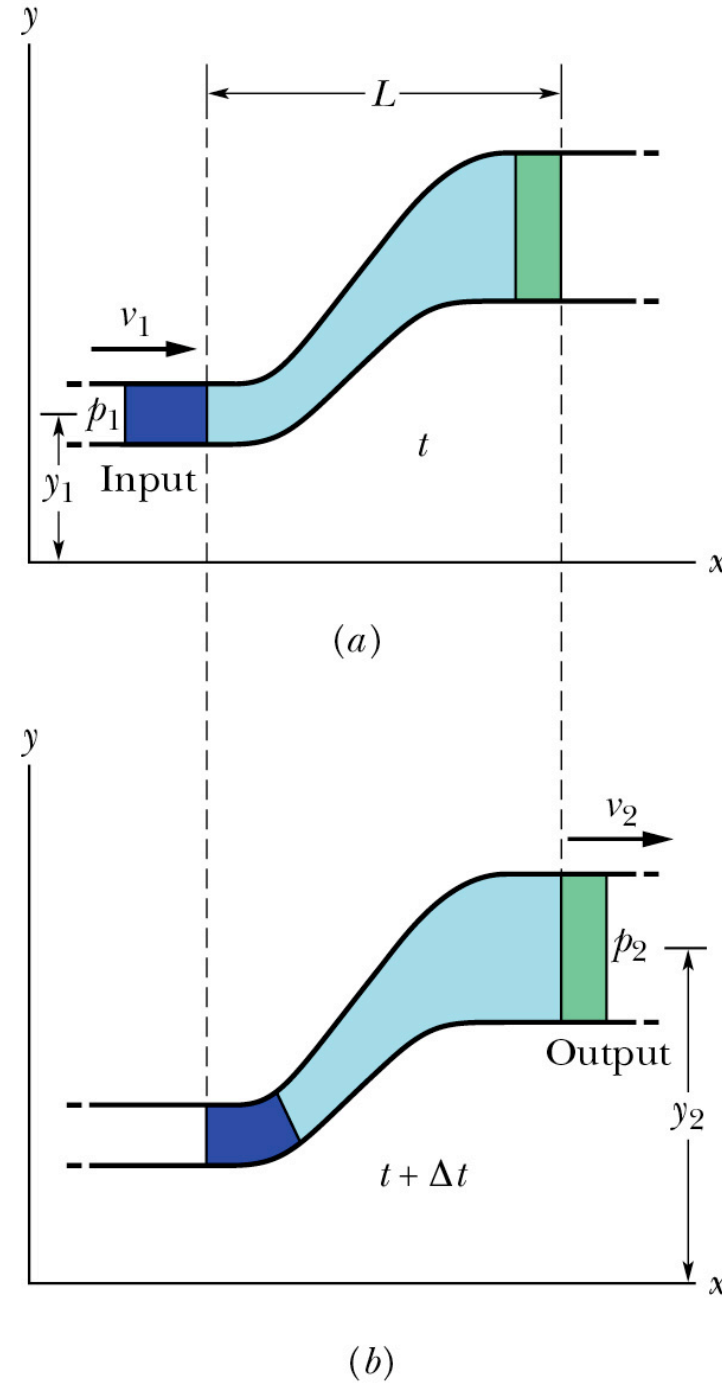
$$p_2 = p_1 + \rho g (y_1 - y_2)$$

y is constant:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

If the speed of a fluid element increases as it travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely

Bernoulli's Equation (proof)



work-kinetic energy theorem: $W = \Delta K$

change in kinetic energy = change in speed during Δt:

$$\begin{aligned}\Delta K &= \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 \\ &= \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)\end{aligned}$$

work done by the gravitational force :

$$\begin{aligned}W_g &= -\Delta m g (y_2 - y_1) \\ &= -\rho g \Delta V (y_2 - y_1)\end{aligned}$$

work done to push the fluid:

$$F\Delta x = (pA)(\Delta x) = p(A\Delta x) = p\Delta V$$

work done on the system is then:

$$\begin{aligned}W_p &= -p_2 \Delta V + p_1 \Delta V \\ &= -(p_2 - p_1)\Delta V\end{aligned}$$

Note that: $W = W_g + W_p = \Delta K$

$$-\rho g \Delta V (y_2 - y_1) - \Delta V (p_2 - p_1) = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

matches Bernoulli's equation

Summary

Density ρ of a fluid at any point $\rho = \frac{\Delta m}{\Delta V}$ or $\rho = \frac{m}{V}$ if it is uniform

Fluid at rest: $p = p_0 + \rho g h$

Archimedes' Principle: $F_b = m_f g$

Bernoulli's equation: $p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant}$