PHYS151 Lecture 13

Ch 13 Equilibrium and Elasticity

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Equilibrium (평형)

Let's consider the following objects:

1) a book resting on a table
2) a hockey puck sliding across a frictionless surface with constant velocity
3) the rotating blades of ceiling fan
4) the wheel of a bicycle that is traveling along a straight path at a constant speed

such objects are in equilibrium and two requirements for equilibrium are then

$$\vec{P} = \text{a constant} \quad \text{and} \quad \vec{L} = \text{a constant}$$

If objects do not move in translation or in rotation, such objects are in static equilibrium and requirements are

$$\vec{P} = 0 \quad \text{and} \quad \vec{L} = 0$$

The Requirements of Equilibrium

Newton's 2nd law in its linear momentum form:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

If the body is in translational equilibrium

$$\vec{F}_{net} = 0$$

(balance of forces)

Newton's 2nd law in its angular momentum form:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

If the body is in rotational equilibrium

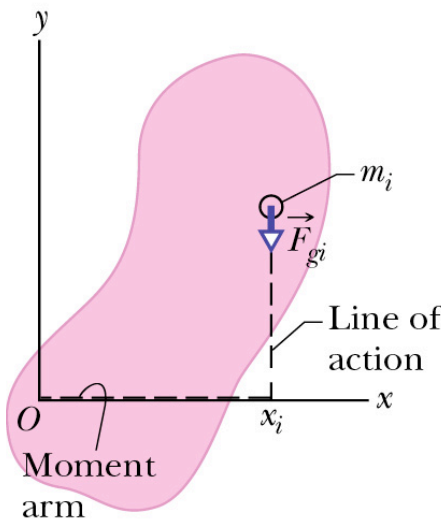
$$\vec{\tau}_{net} = 0$$

Two requirements for a body to be in equilibrium:

1. The vector sum of all the external forces that act on the body must be zero
2. the vector sum of all the external torques that act on the body, measured about any possible point, must also be zero

The Center of Gravity

The gravitational force on a body effectively acts at a single point, called the center of gravity (cog) of the body



(a)

If $\vec{g}$ is the same for all elements of a body, then the body's center of gravity (cog) is coincident with the body's center of mass (com)

Proof:

For an element *i* $\qquad \tau_i = x_i F_{gi}$

the net torque is then $\qquad \tau_{net} = \sum \tau_i = \sum x_i F_{gi}$

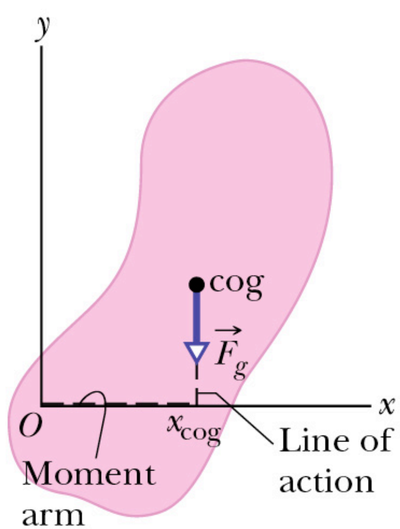
Body as a whole, torque on the body is $\quad \tau = x_{cog} F_g \quad$ or $\quad \tau = x_{cog} \sum F_{gi}$

From the definition of cog, $\quad \tau = \tau_{net} \Rightarrow x_{cog} \sum F_{gi} = \sum x_i F_{gi}$

In terms of mass: $\qquad x_{cog} \sum m_i g_i = \sum x_i m_i g_i$

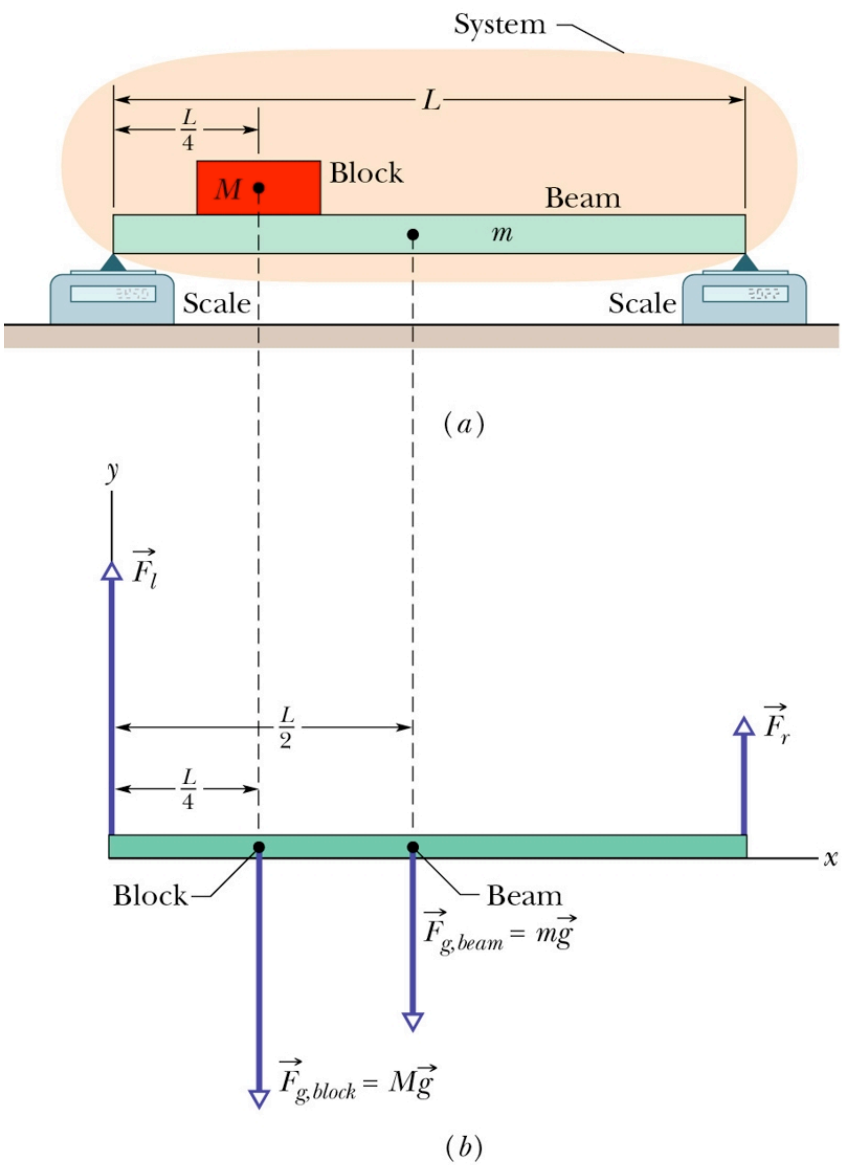
If g_i are all same in everywhere $\quad x_{cog} \sum m_i = \sum x_i m_i \Rightarrow x_{cog} = \frac{1}{M} \sum x_i m_i$

$$x_{cog} = x_{com}$$



(b)

Static Equilibrium : examples



A uniform beam of length *L* and mass *m* = 1.8 kg, is at rest with its ends on two scales. A uniform block, with mass *M* = 2.7 kg, is at rest on the beam, with its center a distance *L*/4 from the beam's left end. What do the scales read?

The system is in static equilibrium

Net force should be zero:

$$F_l + F_r - Mg - mg = 0$$

Net torque should be zero: (we take the left end of the beam as the rotational axis)

$$(0)F_l - (L/4)Mg - (L/2)mg + (L)F_r = 0$$

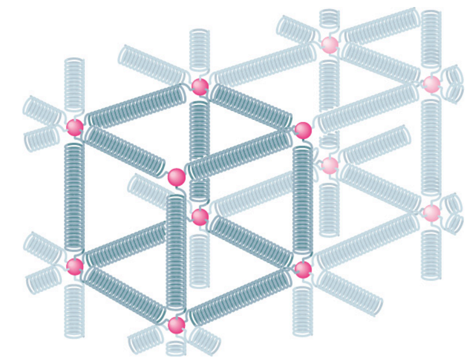
Now we can solve for two F's:

$$F_r = \frac{Mg}{4} + \frac{mg}{2}$$

$$F_l = (M+m)g - F_r$$

Elasticity (탄성)

Rigid body does not deform when forces are applied: true only for perfectly rigid body and elasticity comes in for real life objects

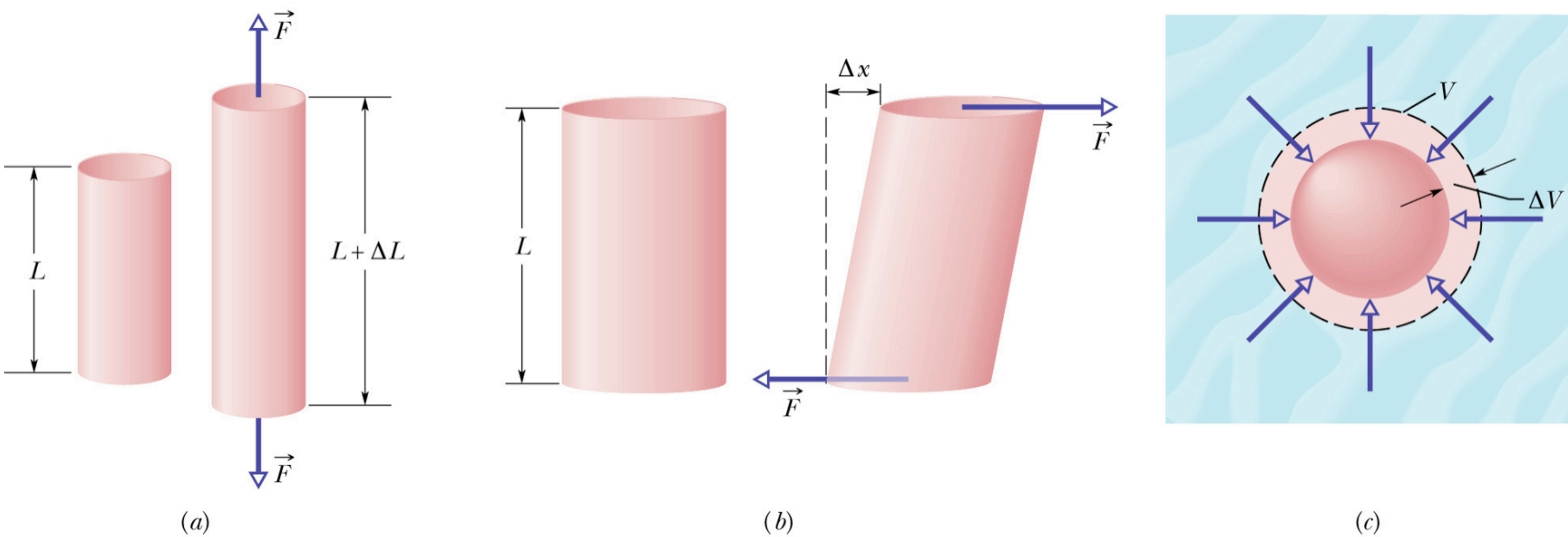


stress : deforming force per unit area

tensile (잡아당기는) stress

shearing (자르는) stress

hydraulic (수력의) stress



Elasticity (탄성)

Tension and Compression

stress is defined as *F/A* (*F* is the force applied perpendicularly to the area *A*)

strain is defined as Δ*L/L* (Δ*L/L* is the fractional change in the length)

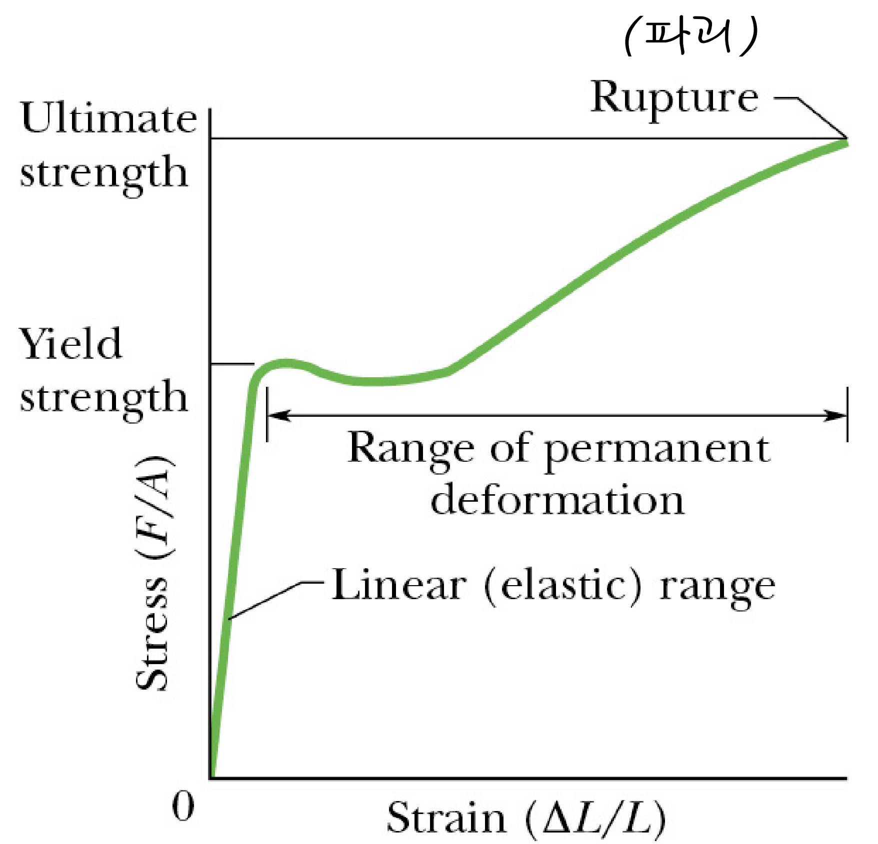
Young's modulus *E*:

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

Shearing

Shearing modulus *G*:

$$\frac{F}{A} = G\frac{\Delta x}{L}$$



Elasticity (탄성)

Hydraulic Stress

bulk modulus *B*:

$$p = B\frac{\Delta V}{V}$$

where *p* is the fluid pressure

ex) Bulk modulus for water: 2.2×10^{9} N/m^2

steel : 16×10^{10} N/m^2

The pressure at the bottom of the Pacific Ocean is 4.0×10^{7} N/m^2

So, fractional compression is 1.8 % for water

0.025 % for steel

Summary

Static Equilibrium: $\vec{F}_{net} = 0$ $\vec{\tau}_{net} = 0$

Center of Gravity: The gravitational force on a body effectively acts at a single point, called the center of gravity (cog) of the body

Tension and Compression $$\frac{F}{A} = E\frac{\Delta L}{L}$$

Shearing $$\frac{F}{A} = G\frac{\Delta x}{L}$$

Hydraulic Stress $$p = B\frac{\Delta V}{V}$$