PHYS151 Lecture 07

Ch 07 Kinetic Energy and Work

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Energy and Work

Energy: a scalar quantity that is associated with a state (I know this is too vague)

Kinetic energy: $$K = \frac{1}{2}mv^2$$

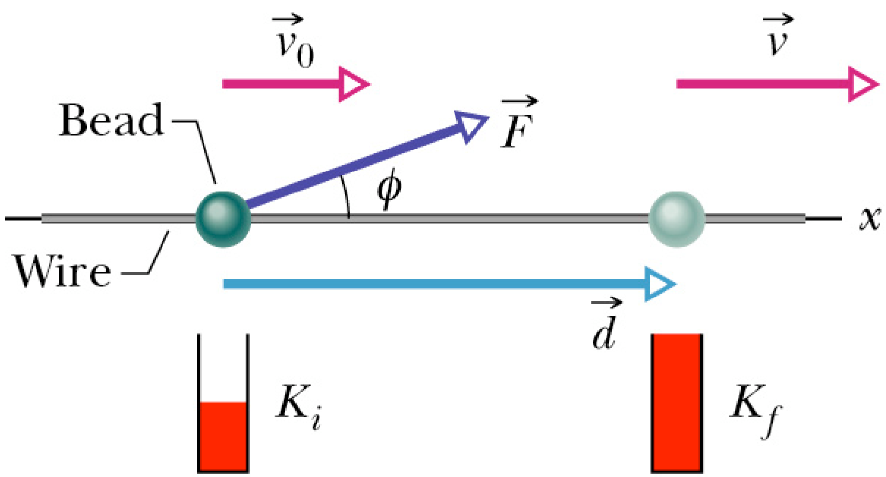
SI unit of kinetic energy (and every other type of energy): Joule (J)

1 Joule = 1 J = 1 kg m^2/s^2

Work is energy transferred to or from an object by means of a force acting on the object

Work and Kinetic Energy

Consider a bead (구슬) that can slide along a frictionless wire, which is stretched along a horizontal x axis



A constant force, $\vec{F}$ directed at an angle ϕ to the wire, accelerates the bead

$$F_x = ma_x$$

$\vec{d}$: displacement of the bead

$\vec{v}_0$: initial velocity

We know from the basic constant-acceleration equations

$$v^2 = v_0^2 + 2a_x d$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$$

The work W done on the bead by the force (the energy transfer due to the force) is

$$W = F_x d \qquad W = Fd\cos\phi \qquad W = \vec{F}\cdot\vec{d}$$

Work-Kinetic energy Theorem

A force does positive (negative) work if it has a vector component in the same (opposite) direction as the displacement

SI unit for work: Joule (same as kinetic energy unit)

Work-Kinetic Energy Theorem:

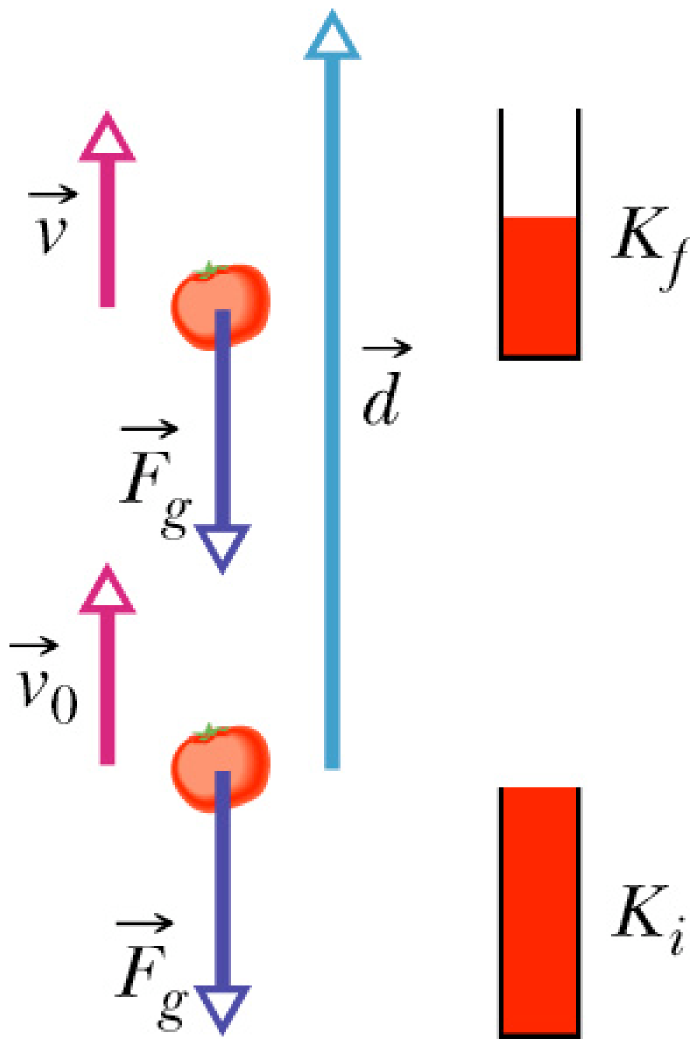
$$\Delta K = K_f - K_i = W$$

$$\left(\begin{array}{c}\text{change in the kinetic}\\ \text{energy of a particle}\end{array}\right) = \left(\begin{array}{c}\text{net work done on}\\ \text{the particle}\end{array}\right)$$

$$K_f = K_i + W$$

$$\left(\begin{array}{c}\text{kinetic energy after}\\ \text{the net work is done}\end{array}\right) = \left(\begin{array}{c}\text{kinetic energy}\\ \text{before the net work}\end{array}\right) + \left(\begin{array}{c}\text{the net}\\ \text{work done}\end{array}\right)$$

Work Done by a Gravitational Force



The work done by the gravitational force:

$$W_g = mgd\cos\phi$$

In the case of the figure left, the object moves upward:

$$W_g = mgd\cos 180^0 = -mgd$$

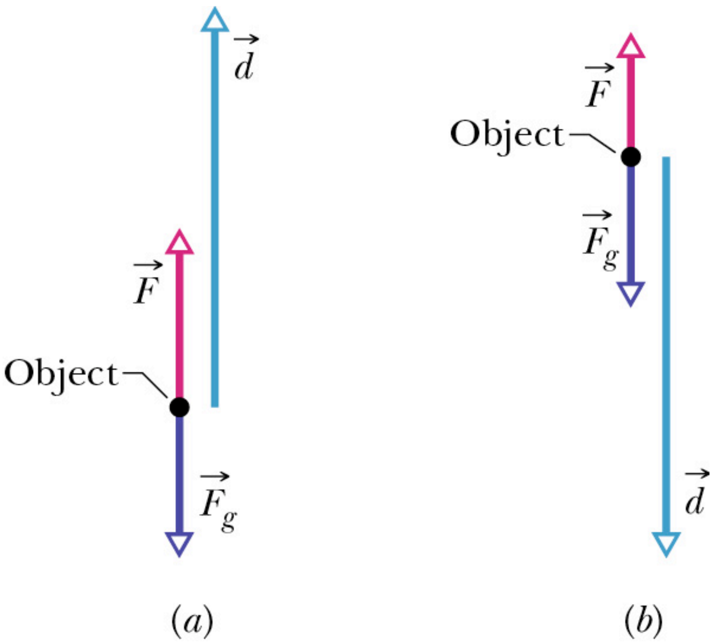
The negative sign indicates that the gravitational force transfers energy (mgd) from the kinetic energy of the object

If the object moves downward:

$$W_g = mgd\cos 0^0 = +mgd$$

The positive sign indicates that the gravitational force transfers energy (mgd) to the kinetic energy of the object

Work Done by a Gravitational Force



Work Done in Lifting and Lowering an Object

Suppose we lift an object by applying a vertical force, our applied force does positive work W_a on the object

The change ΔK in the kinetic energy of the object

$$\Delta K = K_f - K_i = W_a + W_g$$

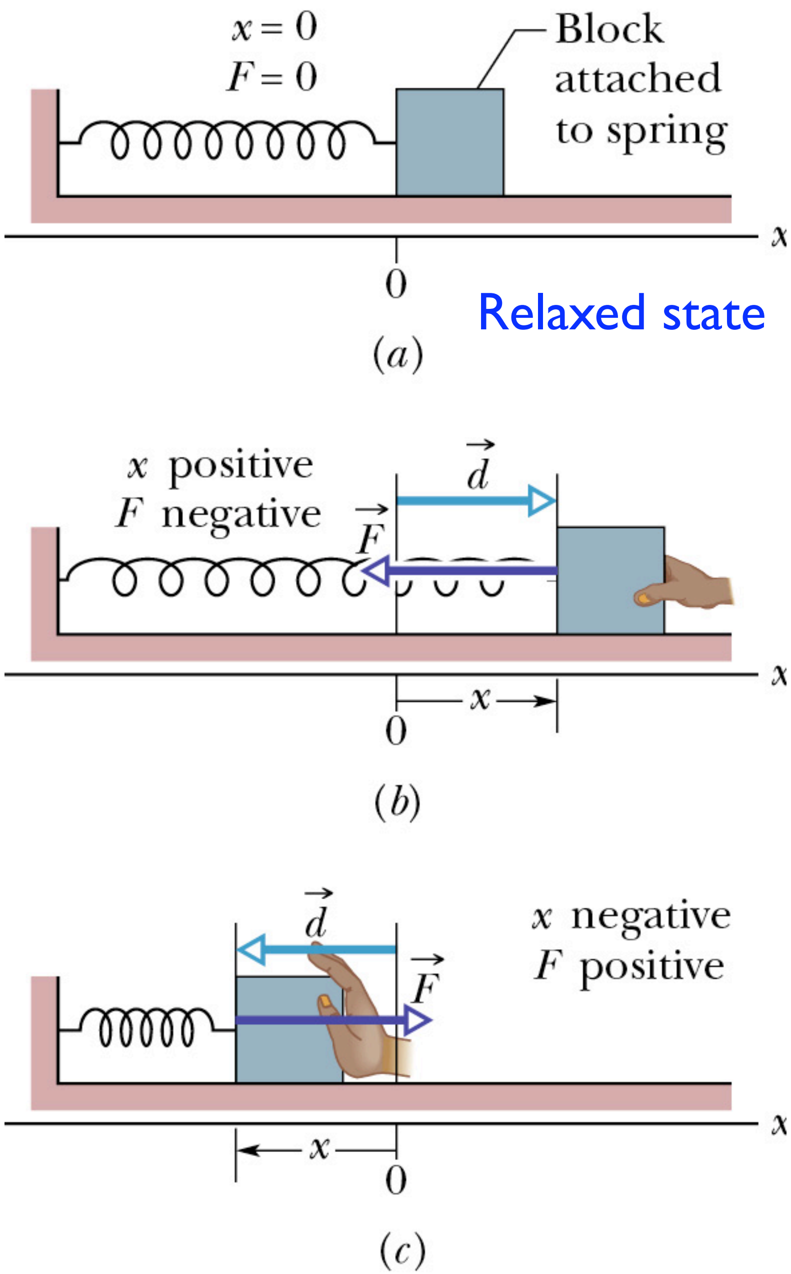
If object is stationary before and after the lift,

$$W_a = -W_g$$

And it becomes

$$W_a = -mgd\cos\phi$$

Work Done by a Spring Force



The Spring Force

$$\vec{F} = -k\vec{d} \quad \text{(Hooke's law)}$$

k : spring constant (force constant)

The negative sign indicates the spring force is always opposite to the displacement

Note: a spring force is a variable force

$$\vec{F} = \vec{F}(x)$$

(This is the first time to see this type of force)

Work Done by a Spring Force

The work done by a spring force

Since the spring force is a variable force, we cannot assume $W = Fd\cos\phi$

The angle ϕ = 0 and the work done within each segment (j) can be written as $F_j \Delta x$

The net work W_s done by spring, from x_i to x_f is the sum of all works:

$$W_s = \sum F_j \Delta x$$

In the limit as each segment goes to zero,

$$W_s = \int_{x_i}^{x_f} F dx$$

Using Hooke's law (F=-kx)

$$\begin{aligned} W_s &= \int_{x_i}^{x_f} (-kx) dx = -k \int_{x_i}^{x_f} x \; dx \\ &= (-\frac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\frac{1}{2}k)(x_f^2 - x_i^2) \end{aligned}$$

We get

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

If x_i=0 and x_f=x,

$$W_s = -\frac{1}{2}kx^2$$

(work by a spring force)

Work Done by a Spring Force

The work done by an applied force

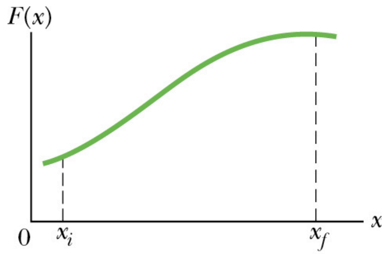
Now suppose that we displace the block along the x axis while continuing to apply a force $\vec{F}_a$ to it. Then

$$\Delta K = K_f - K_i = W_a + W_s$$

If the block is stationary before and after the displacement,

$$W_a = -W_s$$

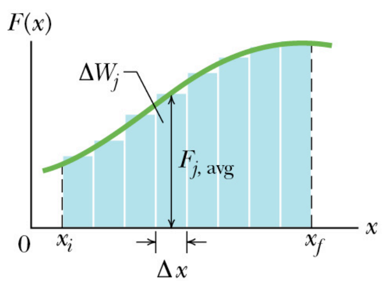
Work Done by a General Variable Force



(a)

Consider one-dimensional variable force (see figure)

$F_{j,avg}$: average value of F(x) within the j-th interval (constant)

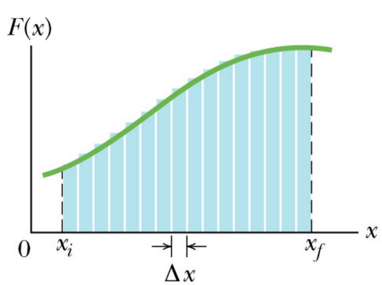


(b)

work done by the force in the j-th interval: $\Delta W_j = F_{j,avg}\Delta x$

The total work done from x_i to x_f is then

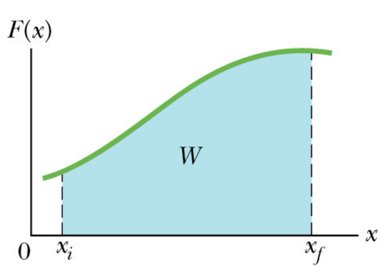
$$W = \sum \Delta W_j = \sum F_{j,avg}\Delta x$$



(c)

We let Δx approach zero:

$$W = \lim_{\Delta x \to 0} \sum F_{j,avg}\Delta x$$



(d)

then we get the integral of the function F(x) (The usual trick in calculus)

$$W = \int_{x_i}^{x_f} F(x)dx$$

(This becomes the area between the F(x) curve and the x axis)

Work Done by a General Variable Force

Three dimensional analysis:

consider now a particle in a three-dimensional force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

Now let the particle move through an incremental displacement

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

Then the increment of work dW is $dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$

Since we know how to calculate this
component by component, we get

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

Work Done by a General Variable Force

Work-Kinetic Energy Theorem with a Variable Force

consider now a particle of mass m, moving along the x axis and acted on a net force F(x) that is directed along that axis

The work done on the particle by this force is:

$$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} ma \ dx$$

Since

$$\begin{aligned} ma \ dx &= m\frac{dv}{dt}dx \\ &= m\frac{dv}{dx}\frac{dx}{dt}dx \\ &= m\frac{dv}{dx}v dx \\ &= mv \ dv \end{aligned}$$

(chain rule here)

We get

$$\begin{aligned} W &= \int_{v_i}^{v_f} mv \ dv = m\int_{v_i}^{v_f} v \ dv \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \end{aligned}$$

or $W = K_f - K_i = \Delta K$

Power

Power: the time rate at which work is done by a force

Average power: $$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous power: $$P = \frac{dW}{dt}$$

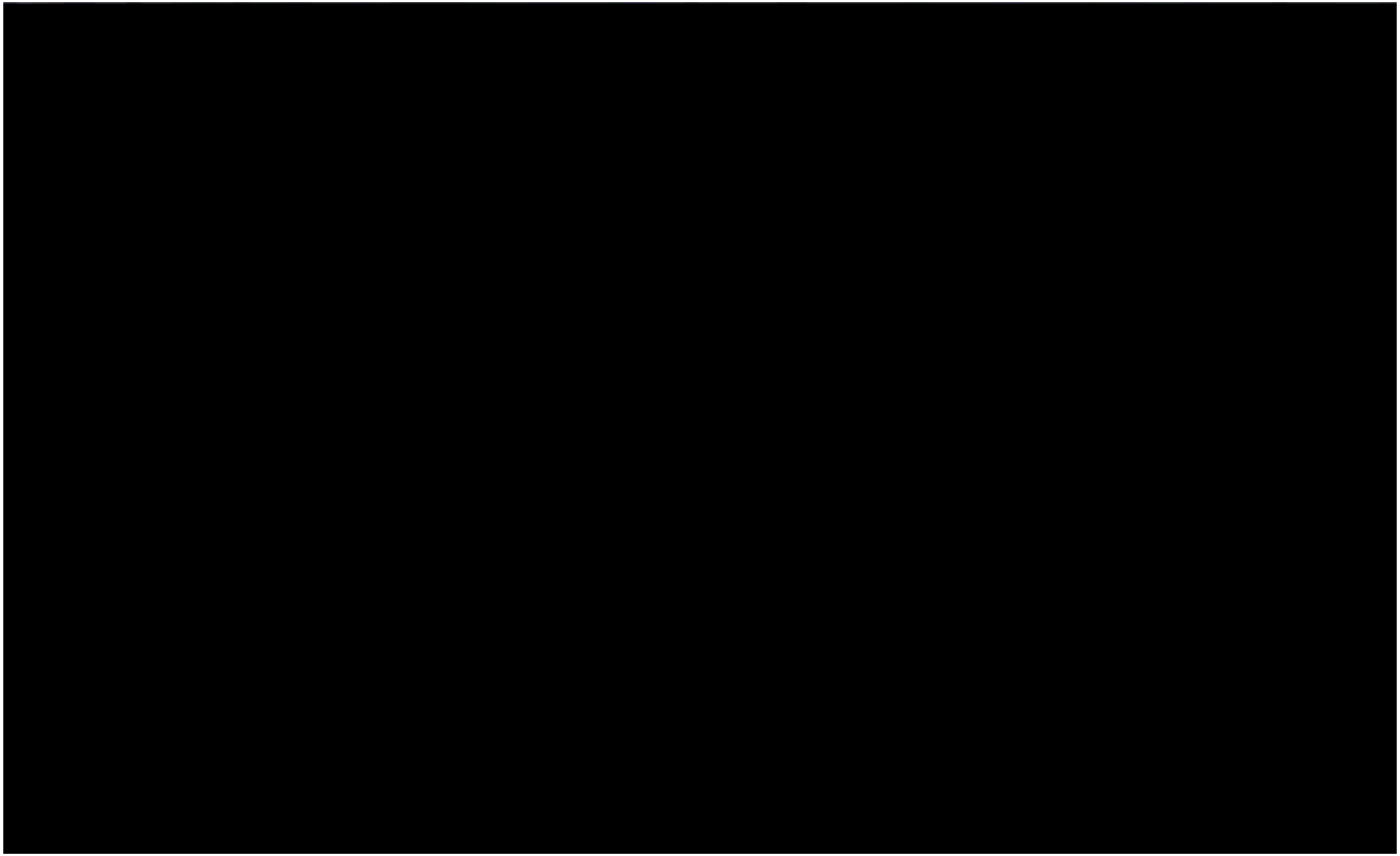
SI unit of power : watt (W)   1 W = 1 J/s

For a particle that is moving along a straight line and is acted on by a constant force directed at some angle φ to that line:

$$\begin{aligned} P &= \frac{dW}{dt} = \frac{F\cos\phi dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right) \\ &= Fv\cos\phi \end{aligned}$$

In general case we write: $$P = \vec{F}\cdot\vec{v}$$

Energy



Summary

Kinetic energy: $$K = \frac{1}{2}mv^2$$

Work: $$W = \vec{F} \cdot \vec{d}$$

Work-Kinetic Energy Theorem: $$\Delta K = K_f - K_i = W$$

The Spring Force: $$\vec{F} = -k\vec{d}$$

Power: $$P = \vec{F} \cdot \vec{v}$$