PHYS151 Lecture 02

Ch 02 Motion Along a Straight Line

Eunil Won
Korea University

Motion

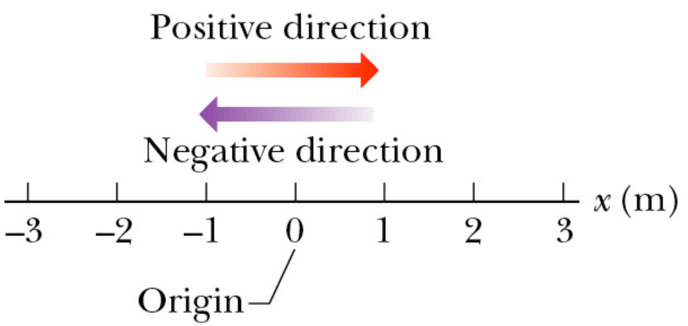
We discuss following properties of motion:

1 The motion is along a straight line only
2 Forces cause motion, but forces will not be discussed until Chapter 5
3 The moving object is a particle

Position and Displacement

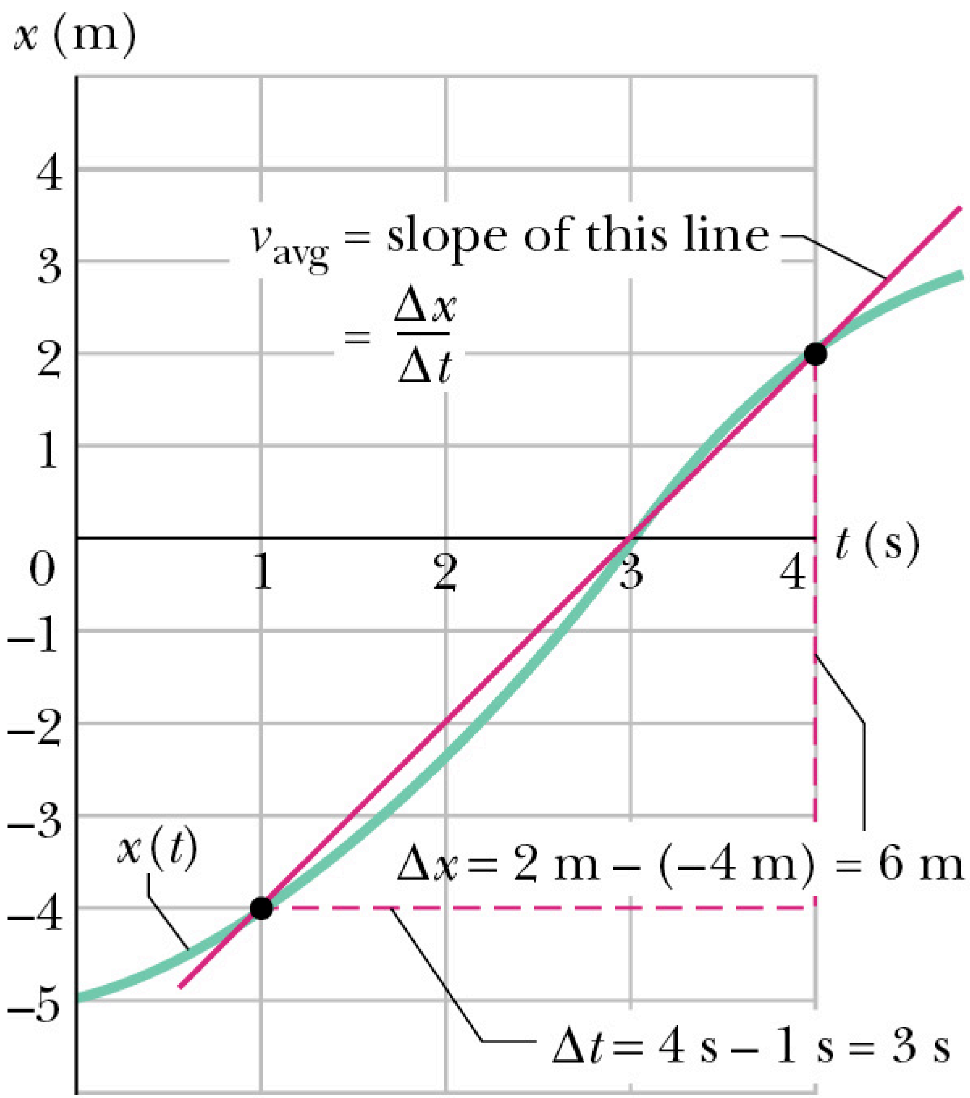
A change from one position x_1 to another position x_2 is called a displacement

$$\Delta x = x_2 - x_1$$



Displacement is a vector quantity: it has both a direction and a magnitude

Average Velocity and Average Speed



Average velocity:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

average velocity becomes the slope in x(t) curve

The average velocity always has the same sign as the displacement ($t_2 > t_1$ always)

Average speed (scalar)

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

Instantaneous Velocity and Speed

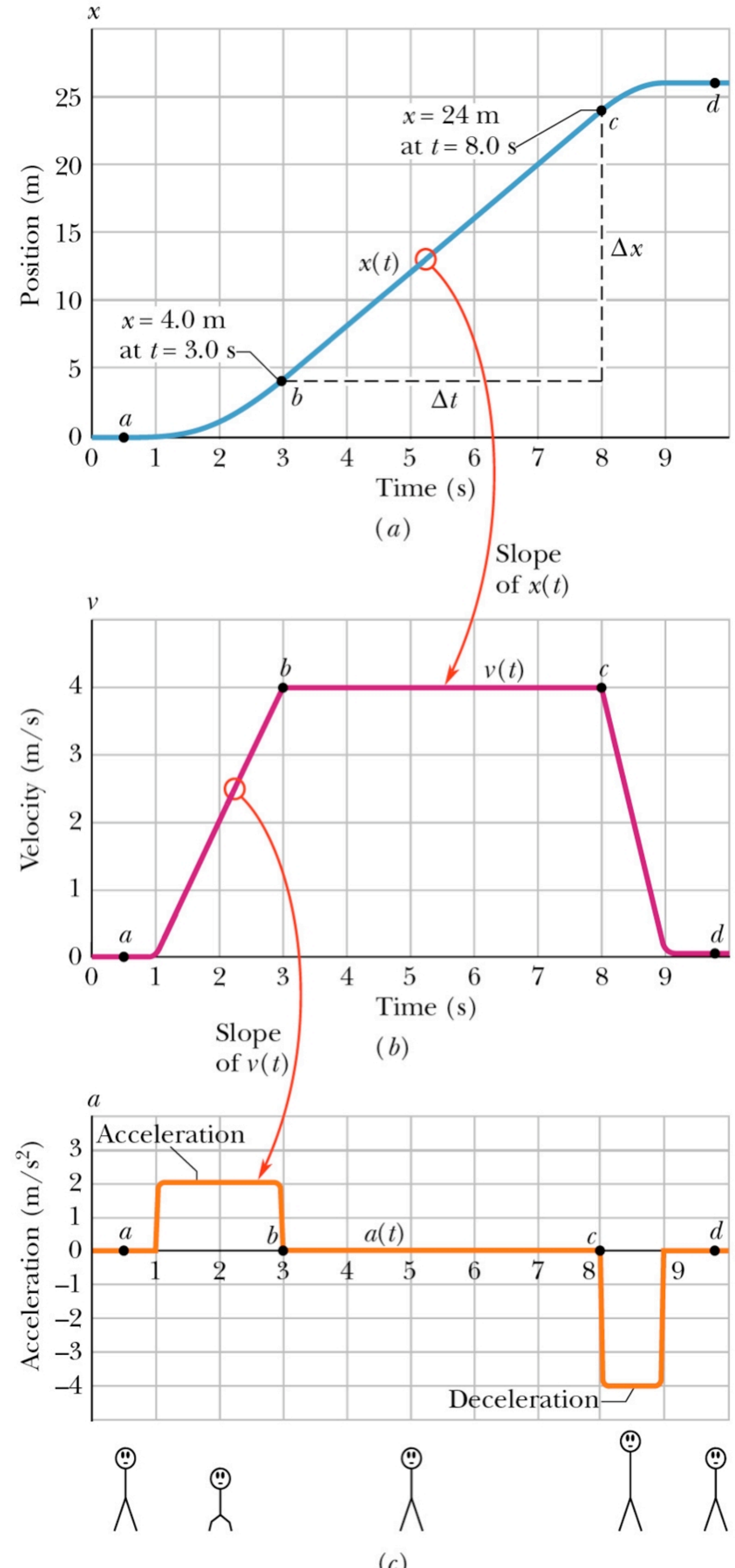
Instantaneous velocity:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

instantaneous velocity becomes the slope in x(t) curve, at any time

Instantaneous speed (scalar)

: magnitude of the instantaneous velocity



Acceleration

:When a particle's velocity changes, the particle is said to undergo acceleration

Average acceleration $$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

(Instantaneous) acceleration $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

SI unit of acceleration: m/s^2

Large acceleration is expressed in terms of g units

1 g = 9.8 m/s^2 (1 g is the magnitude of the acceleration of a falling object)

You experience 4g with this...

Acceleration

: Not sure how strong 4g would be ?

You experience 3g with the roller coaster like this ...

Constant Acceleration: a special case

constant acceleration:

$$a = a_{avg} = \frac{v - v_0}{t - 0}$$

$$v = v_0 + at$$

Similarly, we can rewrite as

$$\begin{aligned} v_{avg} &= \frac{x - x_0}{t - 0} \\ x &= x_0 + v_{avg} t \\ v_{avg} &= \frac{1}{2}(v + v_0) \\ &= v_0 + \frac{1}{2}at \end{aligned}$$

$$\begin{aligned} x - x_0 &= v_0 t + \frac{1}{2}at^2 \\ v &= v_0 + at \end{aligned}$$

Basic equations for constant acceleration

Constant Acceleration: a special case

Now,

$$\begin{aligned} x - x_0 &= v_0\left(\frac{v-v_0}{a}\right) + \frac{1}{2}a\left(\frac{v-v_0}{a}\right)^2 \\ &= \frac{1}{a}(v_0 v - v_0^2) + \frac{1}{2}a\frac{1}{a^2}(v^2 + v_0^2 - 2vv_0) \\ &= -\frac{1}{2}\frac{v_0^2}{a} + \frac{v^2}{2a} \\ \rightarrow v^2 &= v_0^2 + 2a(x - x_0) \end{aligned}$$

(no t)

or

$$\begin{aligned} x - x_0 &= v_0 t + \frac{1}{2}\frac{v-v_0}{t}t^2 \\ &= \frac{1}{2}(v_0 + v)t \end{aligned}$$

(no a)

or

$$\begin{aligned} x - x_0 &= (v - at)t + \frac{1}{2}at^2 \\ &= vt - \frac{1}{2}at^2 \end{aligned}$$

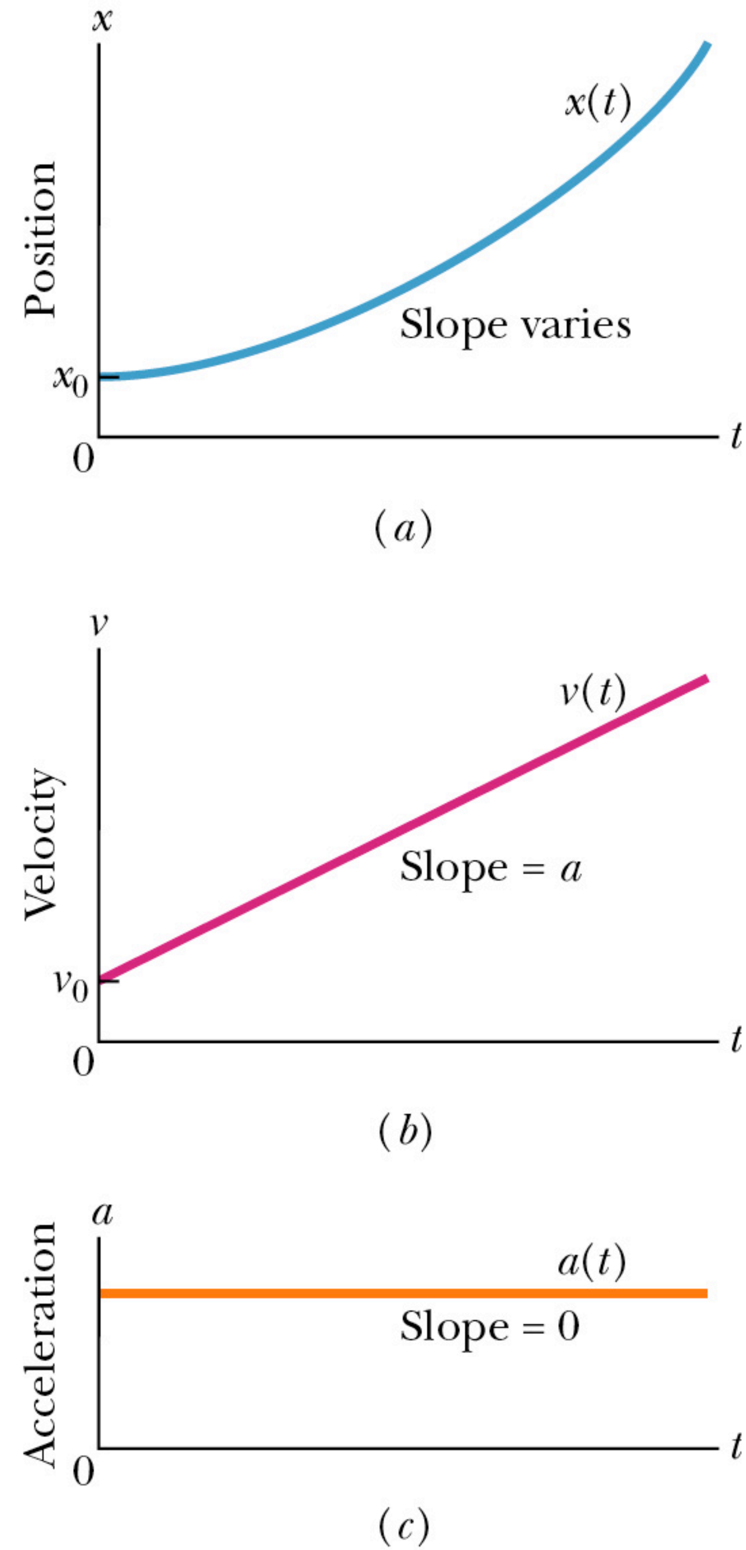
(no v_0)

Constant Acceleration

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

acceleration is constant



Another Look at Constant Acceleration

We re-write the definition of acceleration and take indefinite integral of both sides:

$$
\begin{aligned}
dv &= adt \\
\int dv &= \int adt
\end{aligned}
$$

Acceleration is constant so the integration is trivial. We let $v=v_0$ at $t=0$, that gives $v_0=C$

$$
\begin{aligned}
\int dv &= a\int dt \\
v &= at + C
\end{aligned}
$$

So we get the equation: $v = v_0 + at$

Similarly, we re-write the definition of the velocity:

$$
\begin{aligned}
dx = vdt \rightarrow \int dx &= \int vdt = \int (v_0 + at)dt \\
&= v_0\int dt + a\int tdt \\
&= v_0 t + \frac{1}{2}at^2 + C'
\end{aligned}
$$

We let $x=x_0$ at $t=0$, that gives $x_0=C'$

$$
x - x_0 = v_0 t + \frac{1}{2}at^2
$$

Free-Fall Acceleration

When an object is tossed up or down, it accelerates downward at a certain constant rate (neglecting effects of air on its flights)

magnitude: $g = 9.8\ m/s^2$

free fall acceleration is negative (downward):

$a = -g = -9.8\ m/s^2$

Summary

Displacement: $$\Delta x = x_2 - x_1$$

Average velocity: $$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Instantaneous velocity: $$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Average acceleration $$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

(Instantaneous) acceleration $$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$