PHY604: Final Exam

December 13, 2004

1. Draw all possible Feynman diagrams for following processes below. Consider tree level diagrams only. Indicate the direction of the time in your diagrams (I used the upward direction as the time in the class).

(a) $e^+e^- \to \mu^+\mu^-$ (μ-pair production)
(b) $e^-\mu^- \to e^-\mu^-$
(c) $e^+e^- \to e^+e^-$ (Bhabha scattering)
(d) $e^-e^- \to e^-e^-$ (Moller scattering)
(e) $e^+e^- \to \gamma\gamma$ (pair annihilation)

2. Unless stated explicitly, calculate all in the CM frame.

(a) A particle of mass M decays to a particle of mass m and a massless particle. Find the energies and momenta of the two final particles.

(b) A particle of mass M decays to two particles, one with mass m_1 and the other with mass m_2. Find the energies and momenta of the two final particles.

(c) A particle of mass M decays to three massless particles. Let variables $x_i = 2E_i/M$ parametrize the energies of the final particles. Show that $\sum_i x_i = 2$. Find the kinematically allowed region for (x_1, x_2).

(d) A particle of mass M decays to two massless particles (particles 1 and 2) and one particle of mass m (particle 3). Using the definition of x_i introduced in (c), find the kinematically allowed region for (x_1, x_2). Show that the boundary in this case formed from a straight line and a hyperbola.

3. Evaluate

$$I = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{\omega - (E - i\Gamma)}$$

where E and Γ are real and positive. t can be either positive or negative, so write the results for both cases.

4. Let's consider the a three body decay in the CM frame: $P^\mu = (P, \vec{0})$ and assume m_1, m_2, m_3 are the mass values for three particles in the final state. Let $s = P^2$ and $x_i = 2E_i/\sqrt{s}$, for i = 1,2,3.

(a) Show that the invariant masses $m_{ij}^2 = (p_i + p_j)^2$ are

$$m_{ij}^2 = s(1 - x_k) + m_k^2$$

where i, j, k are non-equal values among 1,2,3. A 2-dimensional plot of m_{ij}^2 vs m_{jk}^2 is called the Dalitz plot.

(b) Show that, in the CM frame, $\vec{p_1}$, $\vec{p_2}$, $\vec{p_3}$ lie in a plane. Express cosines of the angle between any two particles in this plane as functions of x_i, s and m_i.

(c) The final state has 5 degrees of freedom. Explain why that is.

5. A particle of mass M is at rest. It decays to two massive particles m_j and m_k. Assume the particle j is unstable and it decays to two massive particles (particle "1" and particle "2") after a finite amount of time. with the same mass values (For example, $J/\psi \to e^+e^-$). Show that this resonant particle j populates in a straight line in the Dalitz plot of m_{ik}^2 vs m_{2k}^2 . Derive the formula for the straight line.