PHYS152
Lecture 17

Ch 39 Photons and Matter Waves

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The Photon, the Quantum of Light

In 1905, Einstein proposed: electromagnetic radiation is quantized and exists in elementary amounts (quanta) called photons

$$f = \frac{c}{\lambda}$$

The quantum of a light wave of frequency f has energy: E = hf

(energy of single photon)

h: Planck constant: h = 6.63 x 10^{-34} J s = 4.14 x 10^{-15} eV s

ex) A lamp with 100 W power (wavelength=590 nm). How many photons are emitted per second?

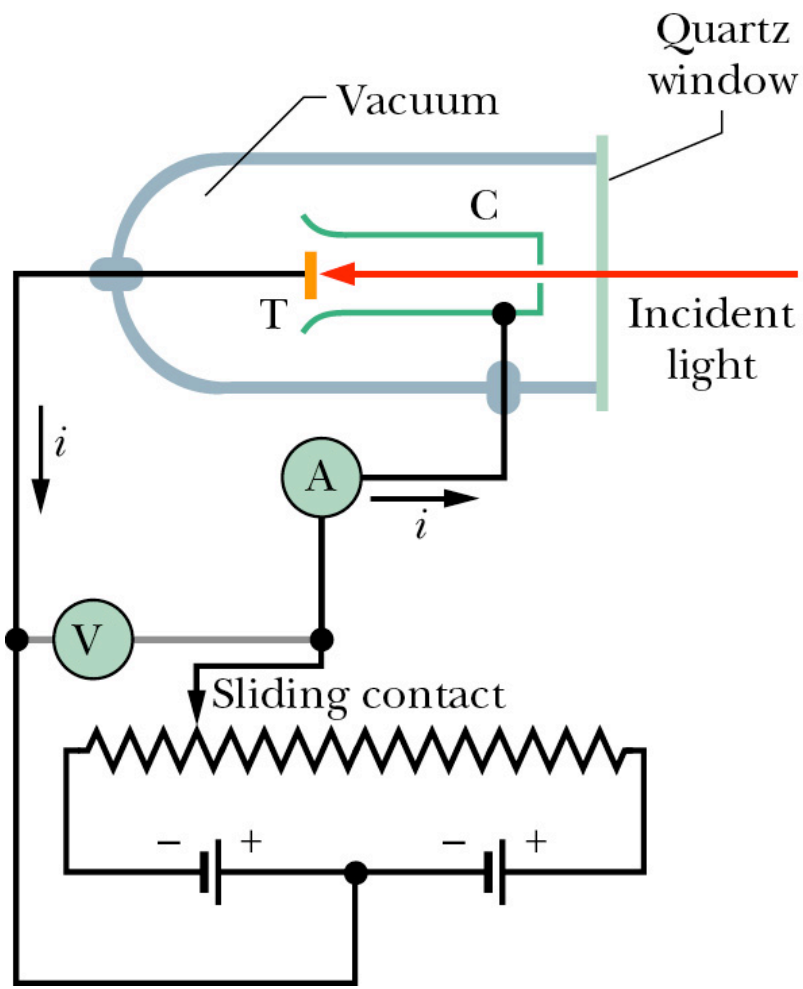
# of photons per second = power / hf = power x c / h x wavelength

$$= \frac{(100\ W)(590 \times 10^{-9} m)}{(6.63 \times 10^{-34} J \cdot s)(3.0 \times 10^{8} m/s)}$$

$$= 2.97 \times 10^{20} photons/s$$

The Photoelectric Effect

If a beam of light is directed onto a clean metal surface, the light cause electrons to leave that surface



First photoelectric experiment

1) incident light causes current

2) apply potential difference V : collector C is slightly negatively charged

3) At certain V, there will be no current

$V = V_{stop}$ (stopping potential)

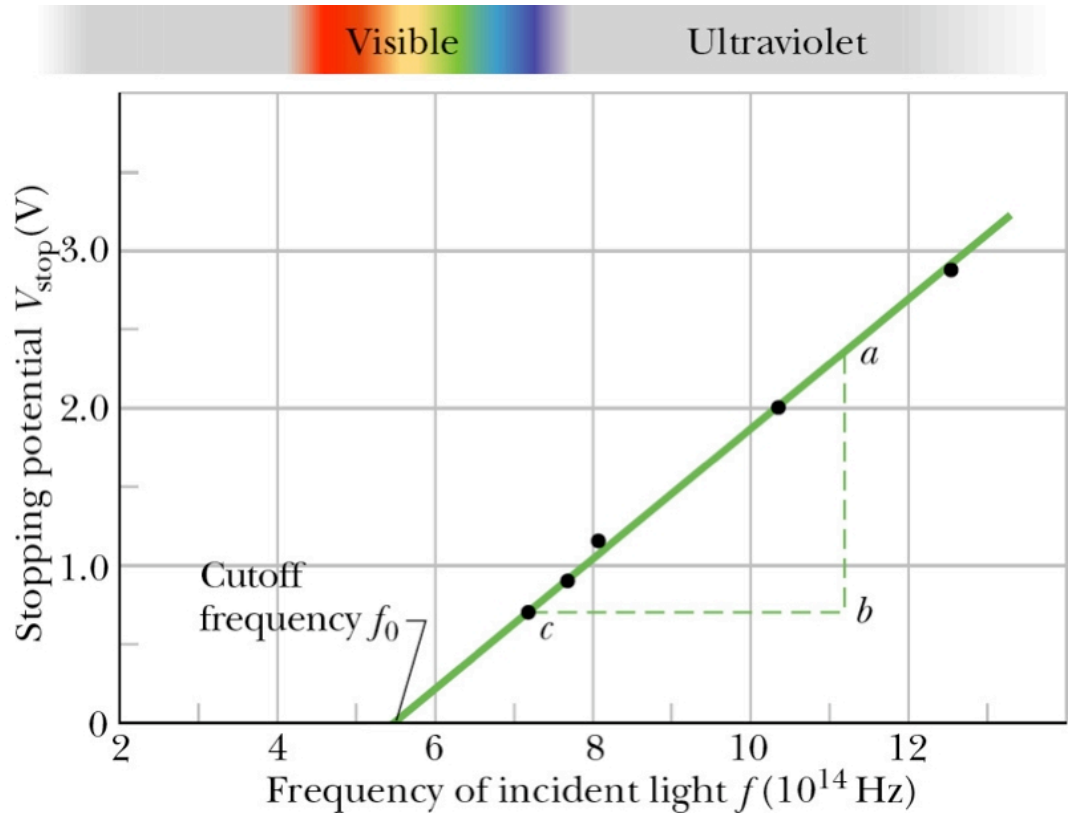
K_{max} : the kinetic energy of most energetic electrons

$$K_{max} = eV_{stop}$$

K_{max} does not depend on the intensity of the light source (inconsistent with wave nature)

The Photoelectric Effect

2nd Photoelectric Experiment: now we vary the frequency of the incident light and measure V_{stop}



Photoelectric effect does not occur below a certain cutoff frequency f_0

$$\lambda_0 = \frac{c}{f_0}$$ (cutoff wavelength)

To just escape from the target, e- must pick up a certain energy Φ (properties of the target material: work function)

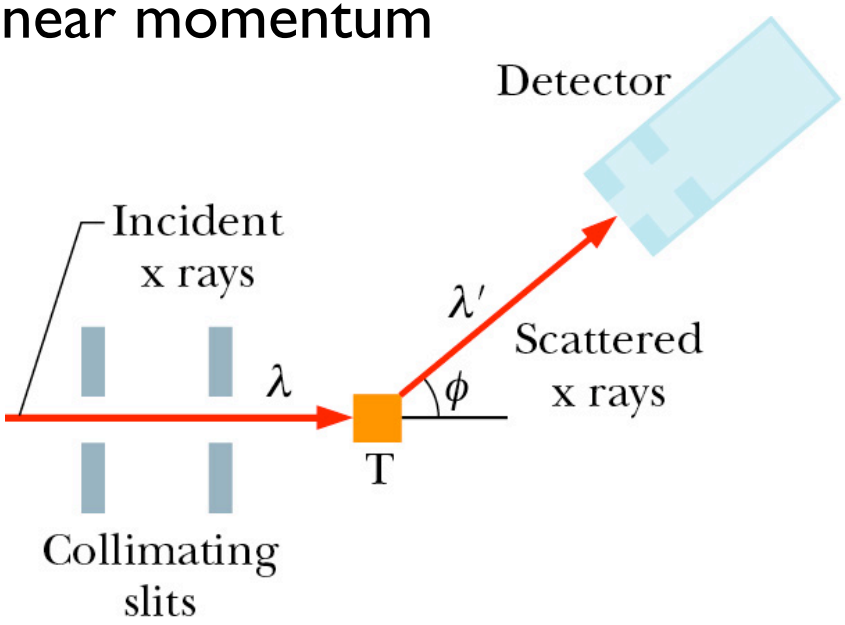
Einstein summed up the photoelectric experiments as:

$$hf = K_{max} + \Phi$$ (photoelectric equation)

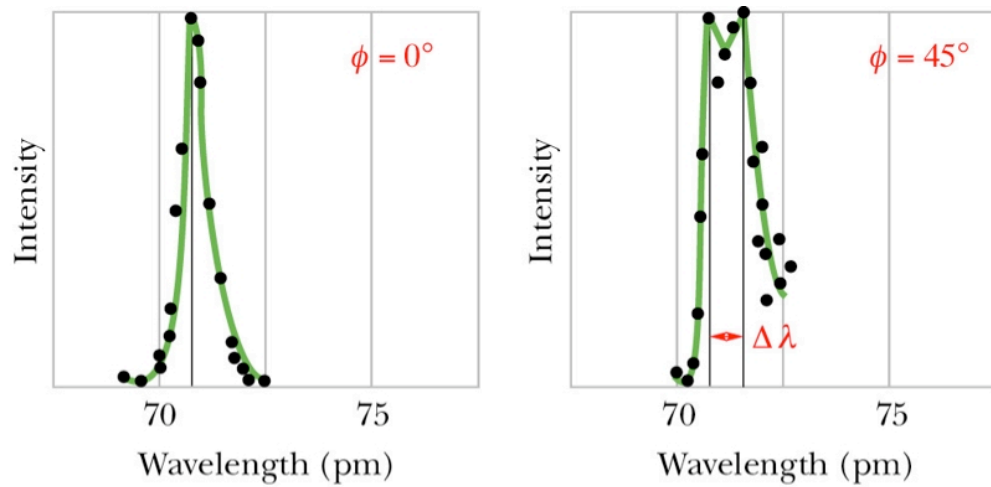
$$V_{stop} = \frac{K_{max}}{e} = \frac{h}{c}f - \frac{\Phi}{e}$$ explains the above plot

Photons have Momentum

In 1916, Einstein extended his concept of light quanta: a quantum of light has linear momentum



$$p = \frac{hf}{c} = \frac{h}{\lambda}$$ (photon momentum)

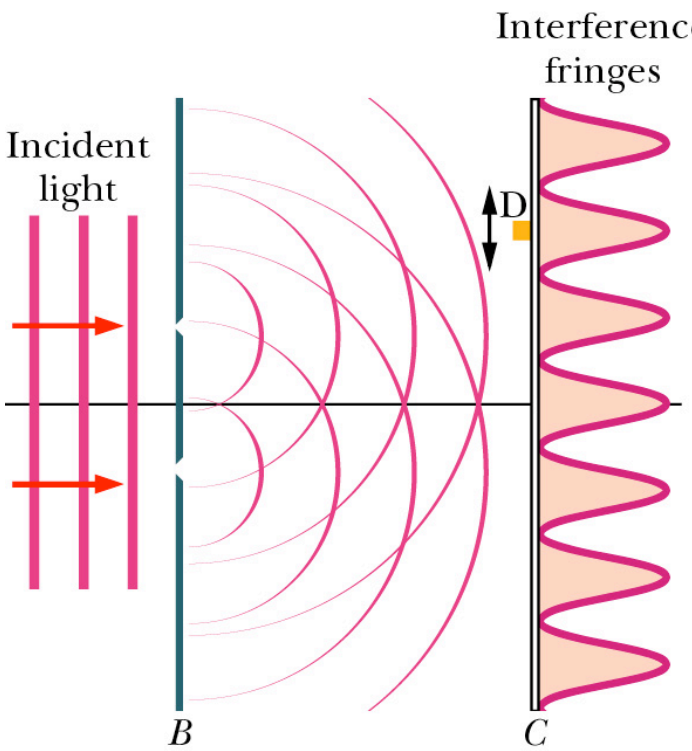


Scattered x rays showed a shift in wavelength (Compton shift)
: a fraction of momentum is transfered

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

Light as a Probability Wave

A fundamental mystery: { Light can be a wave in classical physics
It is emitted and and absorbed as photons (in quantum physics)



Standard Version

: small photon detector tells us relative probability of single photon will be detected
We take a concept of "probability wave"

Single-photon version

: A single-photon version of double-slit experiment (one photon at a time) ->
Astonishingly interference fringes still build up, supporting the probability wave nature

Electrons and Matter Waves

Matter can behave wave?

In 1924, Louis de Broglie suggested matter waves
(A moving matter has wavelength)

$$\lambda = \frac{h}{p}$$

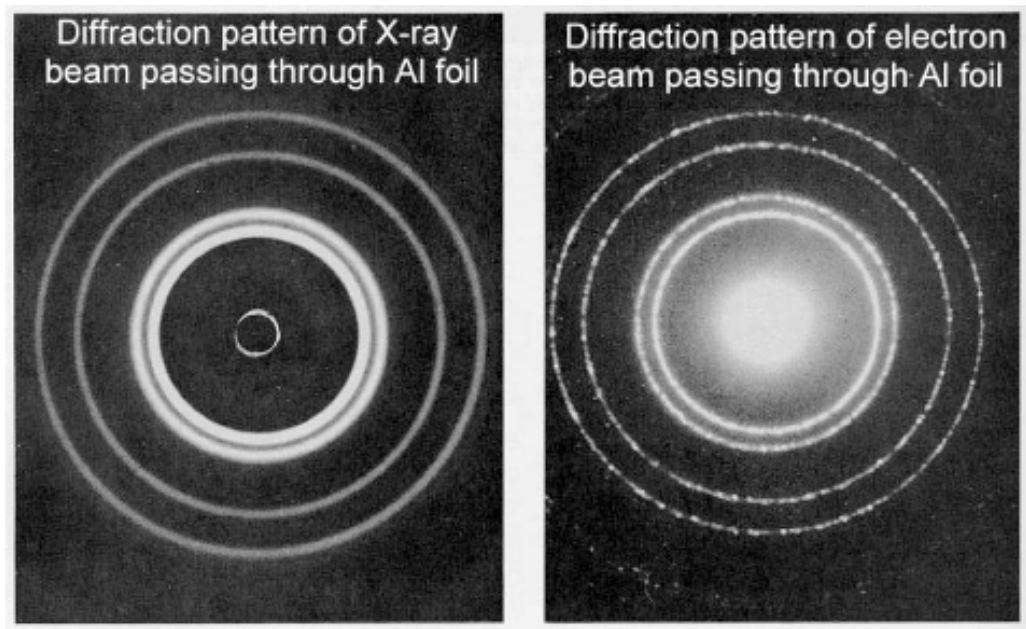
ex) K=120 eV electron

$$\begin{aligned} p &= mv, \qquad K = \frac{1}{2}mv^2 \\ &= m\sqrt{\frac{2K}{m}} = \sqrt{2mK} \\ &= \sqrt{2 \times (9.11 \times 10^{-31} kg)(120eV)(1.6 \times 10^{-19} J/eV)} \\ &= 5.91 \times 10^{-24} kg \cdot m/s \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{6.63 \times 10^{-34} J \cdot s}{5.91 \times 10^{-24} kg \cdot m/s} \\ &= 1.12 \times 10^{-10} m = 112\ pm \end{aligned}$$

ex) Me running v=1m/s

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{6.63 \times 10^{-34} J \cdot s}{60 kg \times 1\ m/s} \\ & 10^{-35}\ m \end{aligned}$$



X-ray and electron diffraction

Wave and Particles

Matter wave: $\Psi(x,y,z,t)$, $i^2 = -1$

$$\Psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$ (space and time is separable in our case)

The probability (per unit time) of detecting a particle in a volume: $|\psi|^2$

Schrodinger Equation : wave equation describes the matter wave

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U_0(x)]\psi = 0$$ (one dimensional case)

For a free particle:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[\frac{1}{2}mv^2]\psi = 0 \qquad \frac{d^2\psi}{dx^2} + \left(2\pi\frac{p}{h}\right)^2\psi = 0$$

$$\underbrace{\frac{d^2\psi}{dx^2} + k^2\psi = 0} \qquad \frac{p}{h} = \frac{1}{\lambda}, \quad \frac{2\pi}{\lambda} = k$$

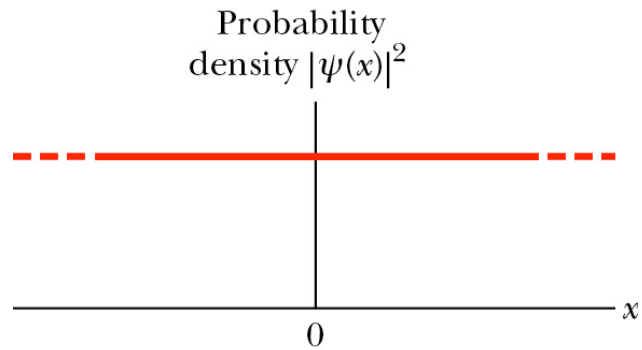
solution to this:

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$\Psi(x,t) = Ae^{i(kx-\omega t)} + Be^{-(ikx+\omega t)}$$

Choose B=0 to get a particle moving +x only

$$|\psi(x)|^2 = constant$$



cannot predict the position of a free particle?

Heisenberg's Uncertainty Principle

The position and the momentum of a particle cannot be measured simultaneously with unlimited precision

$$\Delta x \cdot \Delta p_x \geq \hbar$$

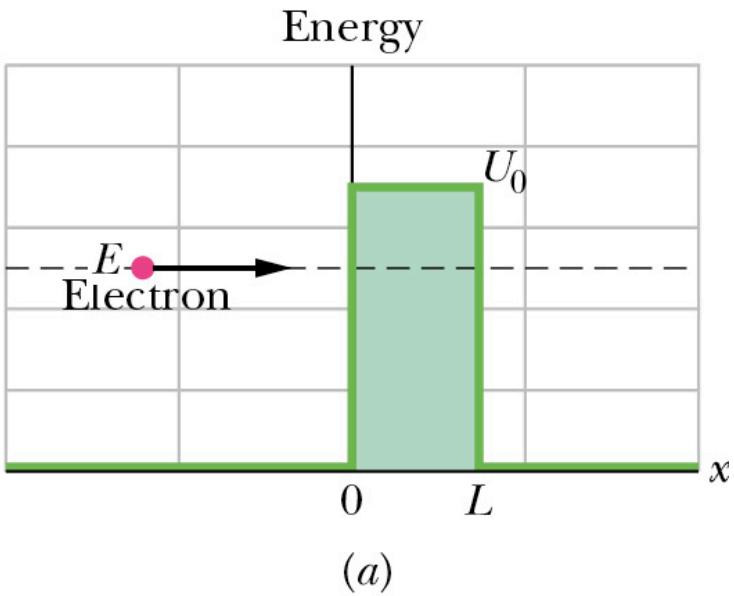
$$\Delta y \cdot \Delta p_y \geq \hbar$$

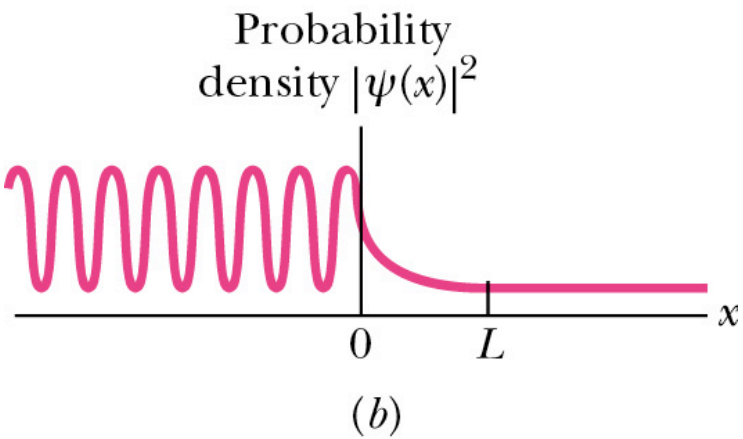
$$\Delta z \cdot \Delta p_z \geq \hbar$$

$$\hbar = \frac{h}{2\pi}$$

Do not think that the particle really has a sharply defined position:
I'm sure you are confused by now :-)

Barrier Tunneling





electron with energy E moving toward to a potential barrier (U_0) when $E<U_0$

classical physics:
the electron is bounced off all the time

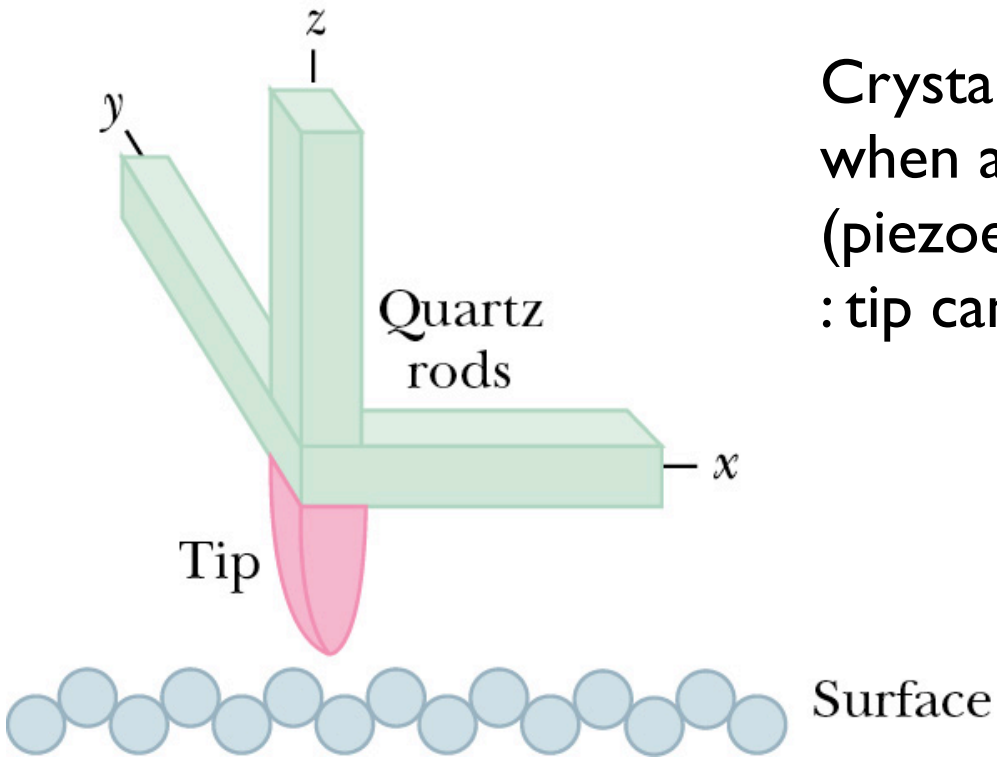
quantum physics:
in some cases the electron penetrates the barrier

Transmission coefficient :
the probability of tunneling of the electron
(If T=0.020, 20 out of 1000 electrons will tunnel through)

$$T \approx e^{-2kL}$$

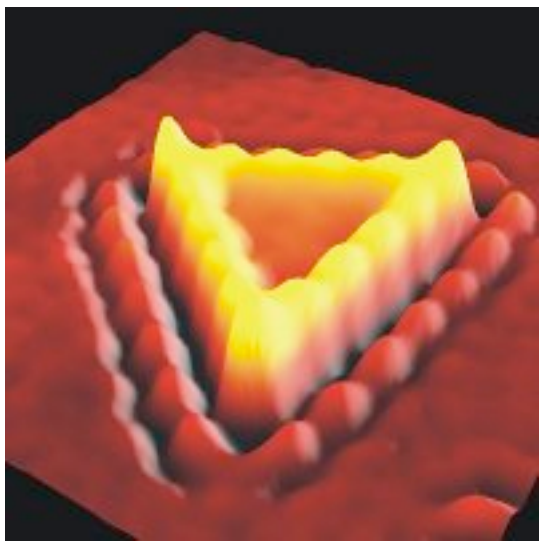
$$k = \sqrt{\frac{8\pi^2 m(U_0 - E)}{h^2}}$$

The Scanning Tunneling Microscope (STM)

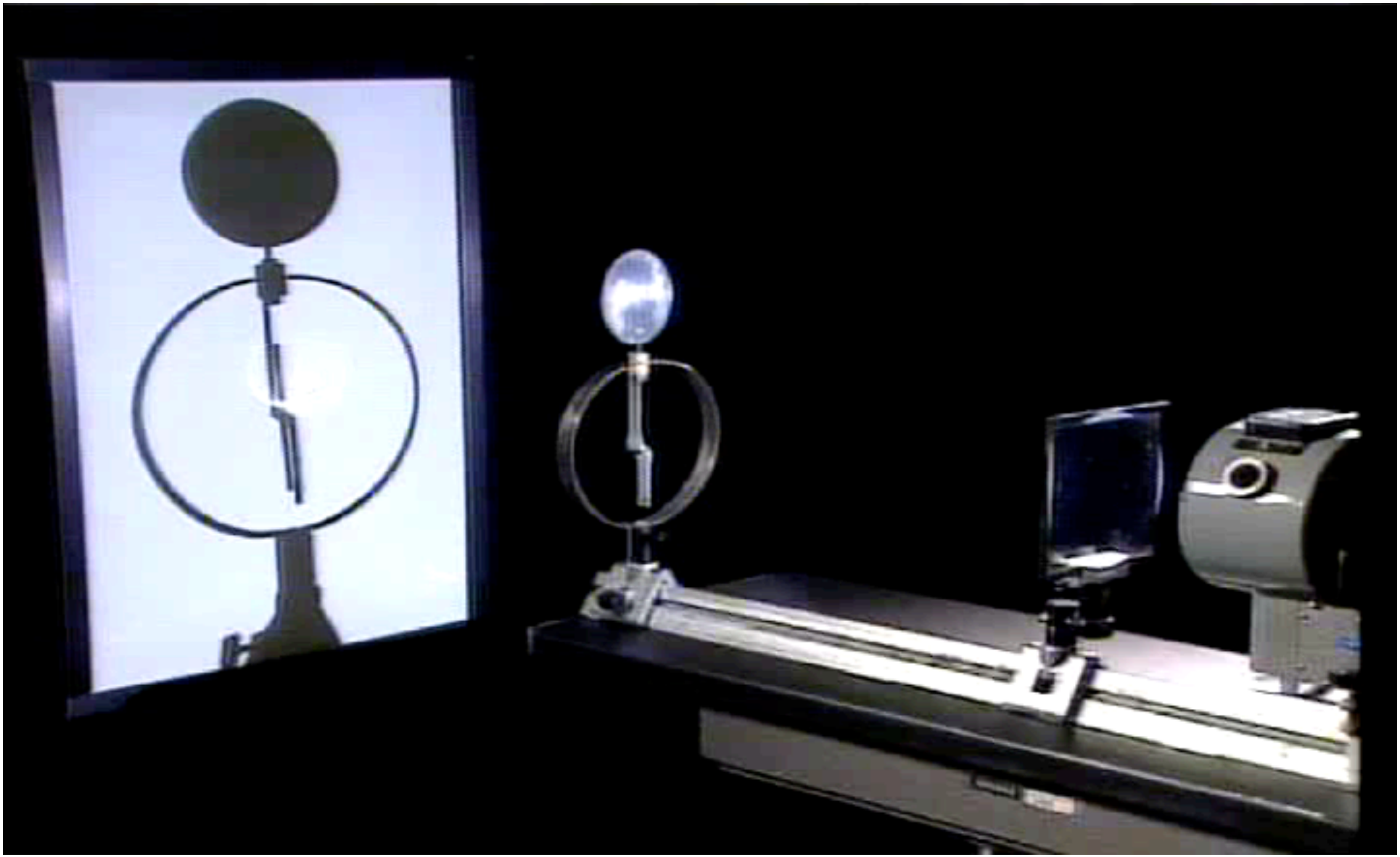


Crystalline quartz changes its dimension when an electric potential is applied (piezoelectricity)
: tip can be moved precisely

Electrons from the sample can tunnel through to the tip
: tunnel current can be measured and used as a microscope (STM)



Photoelectric Effect



Summary

Light Quanta - Photons

Energy E = hf

Momentum $p = \frac{hf}{c} = \frac{h}{\lambda}$

Photoelectric Effect $hf = K_{max} + \Phi$

Compton Shift $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$

Heisenberg's Uncertainty Principle